How can you combine swimming and inline skating to burn 300 Calories?
### APPLICATION: Cross-Training

**Cross-training** involves doing a combination of two or more types of exercise. Since different exercises use different muscle groups, cross-training is a good way to get a well-rounded workout.

**Think & Discuss**

You burn about 12 Calories per minute swimming and about 8 Calories per minute inline skating. You want to do a combination of both activities for a total of 30 minutes and 300 Calories burned.

<table>
<thead>
<tr>
<th>Minutes swimming, ( s )</th>
<th>Minutes inline skating, ( i )</th>
<th>( 12s + 8i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>10</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>15</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>20</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>25</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>30</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

1. Copy the table above. Complete the second column so that \( s + i = 30 \).
2. What does the expression \( 12s + 8i \) represent? Complete this column in your table.
3. How long should you spend doing each activity?

**Learn More About It**

You will find another cross-training combination in Ex. 57 on p. 154.

**APPLICATION LINK** Visit www.mcdougallittell.com for more information about cross-training.
What’s the chapter about?

Chapter 3 is about systems of linear equations and inequalities. In Chapter 3 you’ll learn

• how to solve linear systems in two or three variables by graphing and by using algebraic methods.
• how to write and use linear systems to solve real-life problems.

Are you ready for the chapter?

SKILL REVIEW Do these exercises to review key skills that you’ll apply in this chapter. See the given reference page if there is something you don’t understand.

Check whether the ordered pair is a solution of the given equation or inequality. (Review p. 69; Example 1, p. 108)

1. \( y = \frac{2}{3}x - 4, \ (0, 4) \)  
2. \( x = -3, \ (-3, 1) \)  
3. \( 5x + y = 10, \ (1, 5) \)
4. \( y \geq 0, \ (-4, 5) \)  
5. \( 2x - 3y > 6, \ (6, 2) \)  
6. \( x + y \leq 3, \ (-7, 9) \)

Graph in a coordinate plane. (Review Examples 1–5, pp. 83–85; Examples 2 and 3, p. 109)

7. \( y = \frac{1}{2}x + 1 \)  
8. \( 2x + 5y = 20 \)  
9. \( y = 3 \)
10. \( x \geq -2 \)  
11. \( y < -x \)  
12. \( x - 3y \geq 9 \)

Here’s a study strategy!

Building on Previous Skills

Many of the ideas and skills you will learn in Chapter 3 directly build upon those in Chapter 2. As you study Chapter 3, make a list of important ideas and skills. To help you understand the new material, review the related ideas and skills from Chapter 2. Write these in a second column.
3.1 Solving Linear Systems by Graphing

**GOAL 1** Graphing and Solving a System

A **system of two linear equations** in two variables \(x\) and \(y\) consists of two equations of the following form.

\[
\begin{align*}
Ax + By &= C & \text{Equation 1} \\
Dx + Ey &= F & \text{Equation 2}
\end{align*}
\]

A **solution** of a system of linear equations in two variables is an ordered pair \((x, y)\) that satisfies each equation.

**Example 1** Checking Solutions of a Linear System

Check whether (a) \((2, 2)\) and (b) \((0, -1)\) are solutions of the following system.

\[
\begin{align*}
3x - 2y &= 2 & \text{Equation 1} \\
x + 2y &= 6 & \text{Equation 2}
\end{align*}
\]

**Solution**

a. \(3(2) - 2(2) = 2 \checkmark\) \hspace{1cm} **Equation 1 checks.**

\(2 + 2(2) = 6 \checkmark\) \hspace{1cm} **Equation 2 checks.**

\(\checkmark\) Since \((2, 2)\) is a solution of each equation, it is a solution of the system.

b. \(3(0) - 2(-1) = 2 \checkmark\) \hspace{1cm} **Equation 1 checks.**

\(0 + 2(-1) = -2 \neq 6\) \hspace{1cm} **Equation 2 does not check.**

\(\checkmark\) Since \((0, -1)\) is not a solution of Equation 2, it is not a solution of the system.

**Example 2** Solving a System Graphically

Solve the system.

\[
\begin{align*}
2x - 3y &= 1 & \text{Equation 1} \\
x + y &= 3 & \text{Equation 2}
\end{align*}
\]

**Solution**

Begin by graphing both equations as shown at the right. From the graph, the lines appear to intersect at \((2, 1)\). You can check this algebraically as follows.

\(2(2) - 3(1) = 1 \checkmark\) \hspace{1cm} **Equation 1 checks.**

\(2 + 1 = 3 \checkmark\) \hspace{1cm} **Equation 2 checks.**

\(\checkmark\) The solution is \((2, 1)\).
The system in Example 2 has exactly one solution. It is also possible for a system of linear equations to have infinitely many solutions or no solution.

**EXAMPLE 3**  Systems with Many or No Solutions

Tell how many solutions the linear system has.

a. \(3x - 2y = 6\)
   \(6x - 4y = 12\)

b. \(3x - 2y = 6\)
   \(3x - 2y = 2\)

**SOLUTION**

a. The graph of the equations is the same line. So, each point on the line is a solution and the system has infinitely many solutions.

b. The graphs of the equations are two parallel lines. Because the two lines have no point of intersection, the system has no solution.

---

**CONCEPT SUMMARY**

**NUMBER OF SOLUTIONS OF A LINEAR SYSTEM**

The relationship between the graph of a linear system and the system’s number of solutions is described below.

<table>
<thead>
<tr>
<th><strong>GRAPHICAL INTERPRETATION</strong></th>
<th><strong>ALGEBRAIC INTERPRETATION</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The graph of the system is a pair of lines that intersect in one point.</td>
<td>The system has exactly one solution.</td>
</tr>
<tr>
<td>The graph of the system is a single line.</td>
<td>The system has infinitely many solutions.</td>
</tr>
<tr>
<td>The graph of the system is a pair of parallel lines so that there is no point of intersection.</td>
<td>The system has no solution.</td>
</tr>
</tbody>
</table>
**EXAMPLE 4 Writing and Using a Linear System**

**VACATION COSTS** Your family is planning a 7 day trip to Florida. You estimate that it will cost $275 per day in Tampa and $400 per day in Orlando. Your total budget for the 7 days is $2300. How many days should you spend in each location?

**SOLUTION**

You can use a verbal model to write a system of two linear equations in two variables.

**VERBAL MODEL**

\[
\text{Time spent in Tampa} + \text{Time spent in Orlando} = \text{Total vacation time}
\]

<table>
<thead>
<tr>
<th>Labels</th>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily rate in Tampa \times Time spent in Tampa</td>
<td>Daily rate in Tampa = 275 \quad \text{dollars per day}</td>
<td>Daily rate in Orlando = 400 \quad \text{dollars per day}</td>
</tr>
<tr>
<td>Time spent in Tampa</td>
<td>Time spent in Tampa = \text{x} \quad \text{days}</td>
<td>Time spent in Orlando = \text{y} \quad \text{days}</td>
</tr>
<tr>
<td>Total 7 day budget</td>
<td>Total 7 day budget = 2300 \quad \text{dollars}</td>
<td></td>
</tr>
</tbody>
</table>

**ALGEBRAIC MODEL**

\[
\begin{align*}
\text{Equation 1} & \quad x + y = 7 \\
\text{Equation 2} & \quad 275x + 400y = 2300
\end{align*}
\]

To solve the system, graph each equation as shown at the right.

Notice that you need to graph the equations only in the first quadrant because only positive values of \(x\) and \(y\) make sense in this situation.

The lines appear to intersect at the point \((4, 3)\). You can check this algebraically as follows.

\[
\begin{align*}
4 + 3 & = 7 \quad \checkmark \\
275(4) + 400(3) & = 2300 \quad \checkmark
\end{align*}
\]

The solution is \((4, 3)\), which means that you should plan to spend 4 days in Tampa and 3 days in Orlando.

---

**FOCUS ON APPLICATIONS**

Amusement Business estimates that a family of four would spend about $188.50 at a theme park in Florida.
1. Complete this statement: A(n) ______ of a system of linear equations in two variables is an ordered pair \((x, y)\) that satisfies each equation.

2. How can you use the graph of a linear system to decide how many solutions the system has?

3. Explain why a linear system in two variables cannot have exactly two solutions.

Check whether the ordered pair \((5, 6)\) is a solution of the system.

4. \(-2x + 4y = -14\)
5. \(7x - 2y = 23\)
6. \(x + y = 11\)

Graph the linear system. How many solutions does it have?

7. \(2x - y = 4\)
8. \(14x + 3y = 16\)
9. \(21x - 7y = 7\)

10. **SCHOOL OUTING** Your school is planning a 5 hour outing at the community park. The park rents bicycles for $8 per hour and inline skates for $6 per hour. The total budget per person is $34. How many hours should students spend doing each activity?

Check whether the ordered pair is a solution of the system.

11. \((6, -1)\)
12. \((3, 0)\)
13. \((-2, -8)\)

14. \((-3, -5)\)
15. \((-4, 1)\)
16. \((10, 8)\)

17. \((1, -1)\)
18. \((-2, -7)\)
19. \((0, 2)\)

Graph and check. Graph the linear system and estimate the solution. Then check the solution algebraically.

20. \(2x + y = 13\)
21. \(x + 2y = 9\)
22. \(-2x + y = 5\)

23. \(3x + 4y = -10\)
24. \(2x + y = -11\)
25. \(y = 5x\)

26. \(-x + 3y = 3\)
27. \(2x + y = -2\)
28. \(3x - y = 12\)

29. \(3x - y = 8\)
30. \(y = \frac{1}{6}x - 2\)
31. \(-x + 4y = 10\)

Extra Practice to help you master skills is on p. 943.

**Homework Help**
- **Example 1:** Exs. 11–19
- **Example 2:** Exs. 20–31
- **Example 3:** Exs. 32–52
- **Example 4:** Exs. 54–59
**INTERPRETING A GRAPH** The graph of a system of two linear equations is shown. Tell whether the linear system has *infinitely many solutions*, *one solution*, or *no solution*. Explain your reasoning.

32. ![Graph A]
33. ![Graph B]
34. ![Graph C]

**MATCHING GRAPHS** Match the linear system with its graph. Tell how many solutions the system has.

35. \(2x - y = -5\)  
   \(x + 2y = 0\)

36. \(-2x + 3y = 12\)  
   \(2x - 3y = 6\)

37. \(2x - y = 5\)  
   \(-4x + 2y = -10\)

38. \(x + 5y = -12\)  
   \(x - 5y = 8\)

39. \(-x + 5y = 8\)  
   \(2x - 10y = 7\)

40. \(4x - 7y = 27\)  
   \(-6x - 9y = -21\)

**NUMBER OF SOLUTIONS** Graph the linear system and tell how many solutions it has. If there is exactly one solution, estimate the solution and check it algebraically.

41. \(x = 5\)  
   \(x + y = 1\)

42. \(7x + y = 10\)  
   \(3x - 2y = -3\)

43. \(y = \frac{1}{2}x - 5\)  
   \(y = \frac{1}{2}x + 3\)

44. \(y = -5 - x\)  
   \(x + 3y = -15\)

45. \(\frac{1}{3}x + 7y = 2\)  
   \(\frac{2}{3}x + 14y = 4\)

46. \(-4y = 24x + 4\)  
   \(y = -6x - 1\)

47. \(2x - y = 7\)  
   \(y = 2x + 8\)

48. \(y = \frac{3}{4}x + 3\)  
   \(y = 3x - 6\)

49. \(6x - 2y = -2\)  
   \(3x - 7y = 17\)

50. \(\frac{1}{2}x + 3y = 6\)  
   \(\frac{1}{3}x - 5y = -3\)

51. \(-6x + 2y = 8\)  
   \(y = 3x + 4\)

52. \(\frac{3}{4}x + y = 5\)  
   \(3x + 4y = 2\)
53. **Critical Thinking** Write a system of two linear equations that has the given number of solutions.
   a. one solution  
   b. no solution  
   c. infinitely many solutions

54. **Book Club** You enroll in a book club in which you can earn bonus points to use toward the purchase of books. Each paperback book you order costs $6.95 and earns you 2 bonus points. Each hardcover book costs $19.95 and earns you 4 bonus points. The first order you place comes to a total of $60.75 and earns you 14 bonus points. How many of each type of book did you order? Use the verbal model to write and solve a system of linear equations.

55. **Decoration Costs** You are on the prom decorating committee and are in charge of buying balloons. You want to use both latex and mylar balloons. The latex balloons cost $.10 each and the mylar balloons cost $.50 each. You need 125 balloons and you have $32.50 to spend. How many of each can you buy? Use a verbal model to write and solve a system of linear equations.

56. **Fitness** For 30 minutes you do a combination of walking and jogging. At the end of your workout your pedometer displays a total of 2.5 miles. You know that you walk 0.05 mile per minute and jog 0.1 mile per minute. For how much time were you walking? For how much time were you jogging? Use a verbal model to write and solve a system of linear equations.

57. **Floppy Disk Storage** You want to copy some documents on your friend’s computer. The documents use 6480 kilobytes (K) of disk space. You go to a store and see a sign advertising double-density disks and high-density disks. If you have $6 to spend, how many of each type of disk can you buy to get the disk space you need? Use a verbal model to write and solve a system of linear equations.

58. **Battery Power** Your portable stereo requires 10 size D batteries. You have $25 to spend on 5 packages of 2 batteries each and would like to maximize your battery power. Each regular package of batteries costs $4.25 and each alkaline package of batteries costs $5.50 (because alkaline batteries last longer). How many packages of each type of battery should you buy? Use a verbal model to write and solve a system of linear equations.

59. **Airport Shuttle** A bus station 15 miles from the airport runs a shuttle service to and from the airport. The 9:00 A.M. bus leaves for the airport traveling 30 miles per hour. The 9:05 A.M. bus leaves for the airport traveling 40 miles per hour. Write a system of linear equations to represent distance as a function of time for each bus. Graph and solve the system. How far from the airport will the 9:05 A.M. bus catch up to the 9:00 A.M. bus?
TYPES OF SYSTEMS In Exercises 60–62, use the following definitions to tell whether the system is consistent and independent, consistent and dependent, or inconsistent.

A system that has at least one solution is consistent. A consistent system that has exactly one solution is independent, and a consistent system that has infinitely many solutions is dependent. If a system has no solution, the system is inconsistent.

60. \(-5x + 2y = 12\)  
   \(10x - 4y = -24\)

61. \(-3x - 3y = -6\)  
   \(7x + 4y = 20\)

62. \(2x - y = -12\)  
   \(-6x + 3y = 8\)

63. GEOMETRY CONNECTION Graph the equations \(x + y = 2\), \(-5x + y = 20\), and \(-\frac{5}{7}x + y = -\frac{10}{7}\). What geometric figure do the graphs of the equations form?

   What are the coordinates of the vertices of the figure? Explain the steps you used to find the coordinates.

64. MULTI-STEP PROBLEM You are choosing between two long-distance telephone companies. Company A charges $0.09 per minute plus a $4 monthly fee. Company B charges $0.11 per minute with no monthly fee.

   a. Let \(x\) be the number of minutes you call long distance in one month, and let \(y\) be the total cost of long-distance phone service. Write and graph two equations representing the cost of each company’s service for one month.

   b. Estimate the coordinates of the point where the two graphs intersect. Check your estimate algebraically.

   c. Writing What does the point of intersection you found in part (b) represent? How can it help you decide which long-distance company to use?

65. BUYING A DIGITAL CAMERA The school yearbook staff is purchasing a digital camera. Recently the staff received two ads in the mail. The ad for Store 1 states that all digital cameras are 15% off. The ad for Store 2 gives a $300 coupon to use when purchasing any digital camera. Assume that the lowest priced digital camera is $700. Write and graph two equations that describe the prices at both stores. When does Store 1 have a better deal than Store 2?

### Mixed Review

#### SOLVING EQUATIONS
Solve the equation. Check your solution. (Review 1.3 for 3.2)

66. \(4x + 11 = 39\)  
67. \(\frac{1}{2}x - 10 = 8\)  
68. \(6x - 8 = 3x + 16\)

69. \(-9x - 2 = x + 1\)  
70. \(2(3x - 5) = 7(x + 2)\)  
71. \(10(x + 1) = \frac{1}{2}(x - 18)\)

#### CHECKING SOLUTIONS
Check whether the ordered pairs are solutions of the inequality. (Review 2.6)

72. \(12x + 4y \geq 3\); \((1, -3), (0, 2)\)  
73. \(-x - y \leq -10\); \((-3, -7), (5, 4)\)

74. \(15 > 2x - 2y\); \((10, 3), (-5, 7)\)  
75. \(6x + \frac{1}{2}y \leq -5\); \((2, -6), (-1, 7)\)

#### GRAPHING ABSOLUTE VALUE FUNCTIONS
Graph the function. (Review 2.8)

76. \(y = |x| - 5\)  
77. \(y = |x - 9|\)  
78. \(y = -|x - 8| + 3\)  
79. \(y = |7 - x| + 4\)

3.1 Solving Linear Systems by Graphing
Graphing Systems of Equations

In Lesson 3.1 you learned how to estimate the solution of a linear system by graphing. With a graphing calculator, you can get an answer that is very close to, and sometimes exactly equal to, the actual solution.

**EXAMPLE**

Solve the linear system using a graphing calculator.

\[5x + 3y = -15\]
\[4x - 2y = 45\]

**SOLUTION**

1. Solve each equation for y.

\[5x + 3y = -15\]
\[3y = -5x - 15\]
\[y = -\frac{5}{3}x - 5\]

\[4x - 2y = 45\]
\[-2y = -4x + 45\]
\[y = 2x - \frac{45}{2}\]

2. Enter the equations. It’s a good idea to use parentheses to enter fractions.

3. Using a standard viewing window, graph the equations.

4. Use the *Intersect* feature to find the point where the graphs intersect.

The solution is about \((4.77, -12.95)\).

**EXERCISES**

Solve the linear system using a graphing calculator.

1. \[y = x + 4\]
   \[y = 2x + 5\]
2. \[y = -2x + 13\]
   \[y = 6x - 5\]
3. \[3x - y = 16\]
   \[-5x + 8y = 13\]
4. \[5x + 2y = 6\]
   \[x - 3y = -5\]
5. \[6x + 9y = -13\]
   \[-x + 2y = 10\]
6. \[2x + 8y = -53\]
   \[3x + 4y = 26\]
Combining Equations in a Linear System

**QUESTION**

For a system of two linear equations with exactly one solution, how is the graph of the *sum* of the equations related to the graph of the system?

**EXPLORING THE CONCEPT**

1. Graph the system and label the point of intersection.

\[
\begin{align*}
3x - y &= -5 \\
3x + y &= -1
\end{align*}
\]

2. Add the equations in the system. Graph the resulting equation in the same coordinate plane you used to graph the system.

\[
\begin{align*}
3x - y &= -5 \\
3x + y &= -1
\end{align*}
\]

\[
x = -1
\]

3. Note how the graph of the sum of the equations is related to the graph of the system.

**DRAWING CONCLUSIONS**

1. Repeat the steps above for each system.

   a. \(10x + 4y = 24\)  
   b. \(x - 2y = 2\)  
   c. \(6x - 3y = 27\)  
   d. \(x - y = -3\)  
   e. \(7x + 20y = 0\)  
   f. \(2x - y = -3\)  

\[
\begin{align*}
-6x - 4y &= 8 \\
-x + 4y &= -20 \\
6x &= -6 \\
x &= -1
\end{align*}
\]

2. What seems to be true about the graph of the sum of two equations in a system if the system has exactly one solution?

3. Consider the following general system that has a single solution \((p, q)\).

\[
\begin{align*}
Ax + By &= C \\
Dx + Ey &= F
\end{align*}
\]

Use this system to justify your conclusion from Exercise 2 algebraically.
Solving Linear Systems Algebraically

In this lesson you will study two algebraic methods for solving linear systems. The first method is called *substitution*.

### The Substitution Method

**Step 1** Solve one of the equations for one of its variables.

**Step 2** Substitute the expression from Step 1 into the other equation and solve for the other variable.

**Step 3** Substitute the value from Step 2 into the revised equation from Step 1 and solve.

### Example 1

**The Substitution Method**

Solve the linear system using the substitution method.

\[3x + 4y = -4 \quad \text{Equation 1}\]
\[x + 2y = 2 \quad \text{Equation 2}\]

**Solution**

**1** Solve Equation 2 for \(x\).

\[x + 2y = 2 \quad \text{Write Equation 2.}\]
\[x = -2y + 2 \quad \text{Revised Equation 2.}\]

**2** Substitute the expression for \(x\) into Equation 1 and solve for \(y\).

\[3x + 4y = -4 \quad \text{Write Equation 1.}\]
\[3(-2y + 2) + 4y = -4 \quad \text{Substitute } -2y + 2 \text{ for } x.\]
\[y = 5 \quad \text{Solve for } y.\]

**3** Substitute the value of \(y\) into revised Equation 2 and solve for \(x\).

\[x = -2y + 2 \quad \text{Write revised Equation 2.}\]
\[x = -2(5) + 2 \quad \text{Substitute 5 for } y.\]
\[x = -8 \quad \text{Simplify.}\]

The solution is \((-8, 5)\).

**CHECK** Check the solution by substituting back into the original equations.

\[3(-8) + 4(5) \neq -4 \quad \text{Substitute for } x \text{ and } y.\]
\[\frac{4}{-4} = -4 \quad \text{Solution checks.}\]
\[x + 2y = 2 \quad \text{Original equations.}\]
\[x = -8 + 2(5) \neq 2\]
\[2 = 2 \checkmark\]
CHOOSING A METHOD In Step 1 of Example 1, you could have solved for either \(x\) or \(y\) in either Equation 1 or Equation 2. It was easiest to solve for \(x\) in Equation 2 because the \(x\)-coefficient is 1. In general you should solve for a variable whose coefficient is 1 or \(-1\).

\[
\begin{align*}
  x - 5y &= 11 & \text{Solve for } x. \\
  2x + 7y &= -3
\end{align*}
\]

\[
\begin{align*}
  4x - 2y &= -1 & \text{Solve for } y. \\
  3x - y &= 8
\end{align*}
\]

If neither variable has a coefficient of 1 or \(-1\), you can still use substitution. In such cases, however, the linear combination method may be better. The goal of this method is to add the equations to obtain an equation in one variable.

### THE LINEAR COMBINATION METHOD

**STEP 1** Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables.

**STEP 2** Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.

**STEP 3** Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.

### EXAMPLE 2 The Linear Combination Method: Multiplying One Equation

Solve the linear system using the linear combination method.

\[
\begin{align*}
  2x - 4y &= 13 & \text{Equation 1} \\
  4x - 5y &= 8 & \text{Equation 2}
\end{align*}
\]

**SOLUTION**

1. Multiply the first equation by \(-2\) so that the \(x\)-coefficients differ only in sign.

\[
\begin{align*}
  2x - 4y &= 13 & \times -2 \\
  4x - 5y &= 8 & \text{Write Equation 1.}
\end{align*}
\]

\[
\begin{align*}
  -4x + 8y &= -26 & \text{Substitute } -6 \text{ for } y. \\
  4x - 5y &= 8 & \text{Simplify.}
\end{align*}
\]

2. Add the revised equations and solve for \(y\).

\[
3y = -18 \quad y = -6
\]

3. Substitute the value of \(y\) into one of the original equations and solve for \(x\).

\[
\begin{align*}
  2x - 4y &= 13 & \text{Write Equation 1.} \\
  2x - 4(-6) &= 13 & \text{Substitute } -6 \text{ for } y. \\
  2x + 24 &= 13 & \text{Simplify.}
\end{align*}
\]

\[
x = \frac{-11}{2} \quad \text{Solve for } x.
\]

The solution is \((-\frac{11}{2}, -6)\).

**CHECK** You can check the solution algebraically using the method shown in Example 1. You can also use a graphing calculator to check the solution.
EXAMPLE 3  The Linear Combination Method: Multiplying Both Equations

Solve the linear system using the linear combination method.

\[ 7x - 12y = -22 \]  \hspace{1cm} \text{Equation 1}
\[ -5x + 8y = 14 \]  \hspace{1cm} \text{Equation 2}

**Solution**

*Multiply* the first equation by 2 and the second equation by 3 so that the coefficients of \( y \) differ only in sign.

\[
\begin{align*}
7x - 12y &= -22 & \quad \times 2 \quad & & \quad 14x - 24y &= -44 \\
-5x + 8y &= 14 & \quad \times 3 \quad & & \quad -15x + 24y &= 42
\end{align*}
\]

*Add* the revised equations and solve for \( x \).

\[
\begin{align*}
-x &= -2 \\
x &= 2
\end{align*}
\]

*Substitute* the value of \( x \) into one of the original equations. Solve for \( y \).

\[
\begin{align*}
-5x + 8y &= 14 & \quad \text{Write Equation 2.} \\
-5(2) + 8y &= 14 & \quad \text{Substitute 2 for } x. \\
y &= 3 & \quad \text{Solve for } y.
\end{align*}
\]

\[ \text{The solution is (2, 3). Check the solution algebraically or graphically.} \]

EXAMPLE 4  Linear Systems with Many or No Solutions

Solve the linear system.

\[ a. \ x - 2y = 3 \]
\[ 2x - 4y = 7 \]
\[ b. \ 6x - 10y = 12 \]
\[ -15x + 25y = -30 \]

**Solution**

\[ a. \] Since the coefficient of \( x \) in the first equation is 1, use substitution.

Solve the first equation for \( x \).

\[ x - 2y = 3 \]
\[ x = 2y + 3 \]

Substitute the expression for \( x \) into the second equation.

\[
\begin{align*}
2x - 4y &= 7 & \quad \text{Write second equation.} \\
2(2y + 3) - 4y &= 7 & \quad \text{Substitute } 2y + 3 \text{ for } x. \\
6 &= 7 & \quad \text{Simplify.}
\end{align*}
\]

\[ \text{Because the statement } 6 = 7 \text{ is never true, there is no solution.} \]

\[ b. \] Since no coefficient is 1 or \(-1\), use the linear combination method.

Multiply the first equation by 5 and the second equation by 2.

\[
\begin{align*}
6x - 10y &= 12 & \quad \times 5 & & \quad 30x - 50y &= 60 \\
-15x + 25y &= -30 & \quad \times 2 & & \quad -30x + 50y &= -60
\end{align*}
\]

Add the revised equations.

\[ 0 = 0 \]

\[ \text{Because the equation } 0 = 0 \text{ is always true, there are infinitely many solutions.} \]
**GOAL 2**

**USING LINEAR SYSTEMS IN REAL LIFE**

**EXAMPLE 5 Using a Linear System as a Model**

**CATERING** A caterer is planning a party for 64 people. The customer has $150 to spend. A $39 pan of pasta feeds 14 people and a $12 sandwich tray feeds 6 people. How many pans of pasta and how many sandwich trays should the caterer make?

**SOLUTION**

<table>
<thead>
<tr>
<th>VERBAL MODEL</th>
<th>ALGEBRAIC MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LABELS</strong></td>
<td><strong>EQUATIONS</strong></td>
</tr>
<tr>
<td></td>
<td><strong>MODEL</strong></td>
</tr>
<tr>
<td><strong>Equation 1</strong></td>
<td>People per pan of pasta = 14 (people)</td>
</tr>
<tr>
<td></td>
<td>Pans of pasta = ( P ) (pans)</td>
</tr>
<tr>
<td></td>
<td>People per sandwich tray = 6 (people)</td>
</tr>
<tr>
<td></td>
<td>Sandwich trays = ( S ) (trays)</td>
</tr>
<tr>
<td></td>
<td>People at the party = 64 (people)</td>
</tr>
<tr>
<td></td>
<td>Price per pan of pasta = 39 (dollars)</td>
</tr>
<tr>
<td></td>
<td>Pans of pasta = ( P ) (pans)</td>
</tr>
<tr>
<td></td>
<td>Price per sandwich tray = 12 (dollars)</td>
</tr>
<tr>
<td></td>
<td>Sandwich trays = ( S ) (trays)</td>
</tr>
<tr>
<td></td>
<td>Money to spend on food = 150 (dollars)</td>
</tr>
</tbody>
</table>

Use the linear combination method to solve the system.

**Multiply** Equation 1 by \(-2\) so that the coefficients of \( S \) differ only in sign.

\[
14P + 6S = 64 \quad \times \quad -2 \quad \Rightarrow \quad -28P - 12S = -128
\]

\[
39P + 12S = 150
\]

**Add** the revised equations and solve for \( P \).

\[
11P = 22 \quad \Rightarrow \quad P = 2
\]

**Substitute** the value of \( P \) into one of the original equations and solve for \( S \).

\[
14P + 6S = 64 \quad \text{Write Equation 1.}
\]

\[
14(2) + 6S = 64 \quad \text{Substitute 2 for } P.
\]

\[
28 + 6S = 64 \quad \text{Multiply.}
\]

\[
6 = S \quad \text{Solve for } S.
\]

The caterer should make 2 pans of pasta and 6 sandwich trays for the party.
**GUIDED PRACTICE**

**Vocabulary Check ✓**
1. Complete this statement: To solve a linear system where one of the coefficients is 1 or \(-1\), it is usually easiest to use the **_method**.

**Concept Check ✓**
2. Read Step 3 in the box on page 148. Why do you think it recommends substituting into the revised equation from Step 1 instead of one of the original equations?

3. When solving a linear system algebraically, how do you know when there is no solution? How do you know when there are infinitely many solutions?

**Skill Check ✓**

**Solve the system using the substitution method.**
4. \(x + 3y = -2\)
   \(-4x - 5y = 8\)
5. \(3x + 2y = 10\)
   \(2x - y = 9\)
6. \(-3x + y = -7\)
   \(5x - 2y = 12\)

**Solve the system using the linear combination method.**
7. \(-3x + 2y = -6\)
   \(5x - 2y = 18\)
8. \(5x - 2y = 12\)
   \(-9x - 8y = 19\)
9. \(4x - 3y = 0\)
   \(-10x + 7y = -2\)

10. **BUSINESS** Selling frozen yogurt at a fair, you make $565 and use 250 cones. A single-scoop cone costs $2 and a double-scoop cone costs $2.50. How many of each type of cone did you sell?

**PRACTICE AND APPLICATIONS**

**SUBSTITUTION METHOD** Solve the system using the substitution method.

11. \(2x + 3y = 5\)
   \(x - 5y = 9\)
12. \(-2x + y = 6\)
   \(4x - 2y = 5\)
13. \(-x + 2y = 3\)
   \(4x - 5y = -3\)
14. \(5x + 3y = 4\)
   \(5x + y = 16\)
15. \(4x + 6y = 15\)
   \(-x + 2y = 5\)
16. \(3x - y = 4\)
   \(5x + 3y = 9\)
17. \(\frac{1}{2}x + y = 9\)
   \(7x + 4y = 24\)
18. \(-3x + y = 2\)
   \(8x - 15y = 7\)
19. \(5x + 6y = -45\)
   \(x - \frac{1}{2}y = 8\)
20. \(-x - 4y = -3\)
   \(2x + y = 15\)
21. \(x + 2y = 2\)
   \(7x - 3y = -20\)
22. \(3x - y = 4\)
   \(-9x + 3y = -12\)

**LINEAR COMBINATION METHOD** Solve the system using the linear combination method.

23. \(3x + 5y = -16\)
   \(3x - 2y = -9\)
24. \(3x + 2y = 6\)
   \(-6x - 3y = -6\)
25. \(-6x + 5y = 4\)
   \(7x - 10y = -8\)
26. \(7x - 4y = -3\)
   \(2x + 5y = -7\)
27. \(-9x + 6y = 0\)
   \(-12x + 8y = 0\)
28. \(5x + 6y = -16\)
   \(2x + 10y = 5\)
29. \(21x - 8y = -1\)
   \(9x + 5y = 8\)
30. \(-15x - 2y = -31\)
   \(4x + 6y = 11\)
31. \(\frac{1}{4}x + 5y = 37\)
   \(-4x + 2y = 13\)
32. \(7x + 2y = -3\)
   \(-14x - 4y = 6\)
33. \(6x - y = -2\)
   \(-18x + 3y = 4\)
34. \(-5x + 2y = -10\)
   \(3x - 6y = -18\)

**Example 1**: Exs. 11–22, 35–49
**Examples 2, 3**: Exs. 23–49
**Example 4**: Exs. 11–49
**Example 5**: Exs. 54–62

---

**STUDENT HELP**

**Extra Practice** to help you master skills is on p. 943.

**HOMEWORK HELP**

Example 1: Exs. 11–22, 35–49
Examples 2, 3: Exs. 23–49
Example 4: Exs. 11–49
Example 5: Exs. 54–62
**CHOOSING A METHOD** Solve the system using any algebraic method.

35. \(-5x + 7y = 11\)  
   \(-5x + 3y = 19\)

36. \(x - y = 3\)  
   \(-2x + 2y = -6\)

37. \(2x - 5y = 10\)  
   \(-3x + 4y = -15\)

38. \(-3x + y = 11\)  
   \(5x - 2y = -16\)

39. \(-4x - 6y = 11\)  
   \(6x + 9y = -3\)

40. \(x - 4y = -2\)  
   \(-3x + 8y = 1\)

41. \(2x + 5y = 17\)  
   \(-5x - 7y = -10\)

42. \(-3x + 7y = 6\)  
   \(5x - y = 10\)

43. \(-2x + 3y = 20\)  
   \(4x + 4y = -15\)

44. \(3x - 5y = 20\)  
   \(-11x + 10y = 5\)

45. \(x - y = 17\)  
   \(\frac{1}{2}x - 3y = 1\)

46. \(-3x - 12y = 8\)

47. \(12x + 3y = 16\)  
   \(-36x - 9y = 32\)

48. \(-x + 5y = 17\)  
   \(2x - 10y = -34\)

49. \(\frac{1}{3}x + y = 9\)  
   \(-2x + 2y = -6\)

50. **Writing** Explain how you can tell whether the system has infinitely many solutions or no solution without trying to solve the system.

   a. \(5x - 2y = 6\)  
      \(-10x + 4y = -12\)

   b. \(-2x + y = 8\)  
      \(-6x + 3y = 12\)

**GEOMETRY CONNECTION** Find the coordinates of the point where the diagonals of the quadrilateral intersect.

51.

52.

53.

54. **Breaking Even** You are starting a business selling boxes of hand-painted greeting cards. To get started, you spend $36 on paint and paintbrushes that you need. You buy boxes of plain cards for $3.50 per box, paint the cards, and then sell them for $5 per box. How many boxes must you sell for your earnings to equal your expenses? What will your earnings and expenses equal when you break even?

55. **Home Electronics** To connect a VCR to a television set, you need a cable with special connectors at both ends. Suppose you buy a 6 foot cable for $15.50 and a 3 foot cable for $10.25. Assuming that the cost of a cable is the sum of the cost of the two connectors and the cost of the cable itself, what would you expect to pay for a 4 foot cable? Explain how you got your answer.

56. **Science Connection** Weights of atoms and molecules are measured in atomic mass units (u). A molecule of C₂H₆ (ethane) is made up of 2 carbon atoms and 6 hydrogen atoms and weighs 30.07 u. A molecule of C₃H₈ (propane) is made up of 3 carbon atoms and 8 hydrogen atoms and weighs 44.097 u. Find the weights of a carbon atom and a hydrogen atom.

---

3.2 Solving Linear Systems Algebraically
57. **CROSS-TRAINING** You want to burn 380 Calories during 40 minutes of exercise. You burn about 8 Calories per minute inline skating and 12 Calories per minute swimming. How long should you spend doing each activity?

58. **RENTING AN APARTMENT** Two friends rent an apartment for $975 per month. Since one bedroom is 60 square feet larger than the other bedroom, each person’s rent contribution is based on bedroom size. Each person agrees to pay $3.25 per square foot of bedroom area. Let \( x \) be the area (in square feet) of the larger bedroom, and let \( y \) be the area (in square feet) of the smaller bedroom. Write and solve a system of linear equations to find the area of each bedroom.

**SWIMMING** In Exercises 59—62, use the table below of winning times in the Olympic 100 meter freestyle swimming event for the period 1968–1996.

<table>
<thead>
<tr>
<th>Years since 1968, ( x )</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men’s time (sec), ( m )</td>
<td>52.2</td>
<td>51.2</td>
<td>50.0</td>
<td>49.8</td>
<td>48.6</td>
<td>49.0</td>
<td>48.7</td>
<td></td>
</tr>
<tr>
<td>Women’s time (sec), ( w )</td>
<td>60.0</td>
<td>58.6</td>
<td>55.7</td>
<td>54.8</td>
<td>55.9</td>
<td>54.9</td>
<td>54.6</td>
<td>54.5</td>
</tr>
</tbody>
</table>

59. Use a graphing calculator to make scatter plots of the data pairs \((x, m)\) and \((x, w)\).

60. For each scatter plot, find an equation of the line of best fit. Graph the equations, as shown.

61. Find the coordinates of the intersection point of the lines. Describe what this point represents.

62. **CRITICAL THINKING** Why might a linear model not be appropriate for projecting winning times far into the future?

**QUANTITATIVE COMPARISON** In Exercises 63 and 64, choose the statement that is true about the given quantities.

- A. The quantity in column A is greater.
- B. The quantity in column B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the given information.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
</table>
| 63. The \( x \)-coordinate of the solution of: \[
7x - y = 19 \\
10x + 2y = 34
\] | 3 |
| 64. \(-5\) | The \( y \)-coordinate of the solution of: \[
-2x + 6y = -26 \\
x + 3y = 11
\] |

**Challenge**

65. **CRITICAL THINKING** Find values of \( r, s, \) and \( t \) that produce the solution(s).

\[
-3x - 5y = 9 \\
x + sy = t
\]

- a. no solution
- b. infinitely many solutions
- c. a solution of \((2, -3)\)
**Mixed Review**

**Absolute Value Equations** Solve the equation. (Review 1.7)

66. \(|6x| = 12\)  
67. \(|x + 5| = 3\)  
68. \(|2x - 1| = 7\)  
69. \(|4x + 1| = 5\)  
70. \(|3x - 2| = 8\)  
71. \(|-x + 10| = 14\)

**Writing Equations** Write an equation of the line. (Review 2.4)

72. 73. 74.

**Graphing Inequalities** Graph the inequality in a coordinate plane. (Review 2.6 for 3.3)

75. \(y < 4\)  
76. \(x \geq -2\)  
77. \(3x - y \geq 0\)  
78. \(y < -x + 4\)  
79. \(4x - y < 5\)  
80. \(y \geq -2x - 1\)

81. **Consumer Economics** You plan to buy a pair of jeans for $25 and some T-shirts for $12 each. You have only $60 to spend. Write and solve an inequality for the number of T-shirts you can buy. (Review 1.6 for 3.3)

**Quiz 1**

**Self-Test for Lessons 3.1 and 3.2**

Use a graph to solve the system. (Lesson 3.1)

1. \(y = 2x + 5\) \(y = -2x - 3\)  
2. \(y = -4x + 1\) \(y = x - 4\)  
3. \(-3x + 2y = 4\) \(6x - 4y = 14\)  
4. \(-2x - y = -2\) \(3x - 3y = 15\)  
5. \(y = -x + 5\) \(3x - y = -1\)  
6. \(4x + 5y = -9\) \(x + 3y = -4\)

Tell how many solutions the linear system has. (Lessons 3.1 and 3.2)

7. \(6x + 6y = 3\) \(4x + 4y = 2\)  
8. \(-2x + y = 13\) \(x - 4y = -31\)  
9. \(-5x + 7y = 10\) \(15x - 21y = 22\)  
10. \(3x - 3y = 3\) \(-4x + y = -21\)  
11. \(x - 6y = 6\) \(-3x + 2y = -2\)  
12. \(-4x + 8y = 24\) \(-x + 2y = 6\)

Solve the system using any algebraic method. (Lesson 3.2)

13. \(-2x + 2y = -5\) \(x + y = -5\)  
14. \(-3x + 2y = -6\) \(5x - 2y = 18\)  
15. \(-4x - y = -1\) \(12x + 3y = 3\)  
16. \(-3x - 4y = -2\) \(x + 2y = 3\)  
17. \(3x - 8y = 11\) \(-6x + 16y = -5\)  
18. \(3x - 8y = -7\) \(-5x - 6y = 3\)  
19. **Theater** Tickets for your school’s play are $3 for students and $5 for non-students. On opening night 937 tickets are sold and $3943 is collected. How many tickets were sold to students? to non-students? (Lesson 3.2)
Graphing and Solving Systems of Linear Inequalities

**Goal 1**

Graph a system of inequalities to find the solutions of the system.

**Goal 2**

Use systems of linear inequalities to solve real-life problems, such as finding a person's target heart rate zone in Example 3.

**Why you should learn it**

To solve real-life problems, such as finding out how a moose can satisfy its nutritional requirements in Ex. 58.

**Investigating Graphs of Systems of Inequalities**

The coordinate plane shows the four regions determined by the lines $3x - y = 2$ and $2x + y = 1$. Use the labeled points to help you match each region with one of the systems of inequalities.

- **a.** $3x - y \leq 2$
- **b.** $3x - y \geq 2$
- **c.** $3x - y \geq 2$
- **d.** $3x - y \leq 2$

As you saw in the activity, a system of linear inequalities defines a region in a plane. Here is a method for graphing the region.

**Graphing a System of Linear Inequalities**

To graph a system of linear inequalities, do the following for each inequality in the system:

- Graph the line that corresponds to the inequality. Use a dashed line for an inequality with $<$ or $>$ and a solid line for an inequality with $\leq$ or $\geq$.
- Lightly shade the half-plane that is the graph of the inequality. Colored pencils may help you distinguish the different half-planes.

The graph of the system is the region common to all of the half-planes. If you used colored pencils, it is the region that has been shaded with every color.
3.3 Graphing and Solving Systems of Linear Inequalities

**EXAMPLE 1**

**Graphing a System of Two Inequalities**

Graph the system.

\[
\begin{align*}
y & \geq -3x - 1 \\
y & < x + 2
\end{align*}
\]

**SOLUTION**

Begin by graphing each linear inequality. Use a different color for each half-plane. For instance, you can use red for Inequality 1 and blue for Inequality 2. The graph of the system is the region that is shaded purple.

\[\text{Shade the half-plane on and to the right of } y = -3x - 1 \text{ red.}\]
\[\text{Shade the half-plane below } y = x + 2 \text{ blue.}\]

You can also graph a system of three or more linear inequalities.

**EXAMPLE 2**

**Graphing a System of Three Inequalities**

Graph the system.

\[
\begin{align*}
x & \geq 0 \\
y & \geq 0 \\
4x + 3y & \leq 24
\end{align*}
\]

**SOLUTION**

Inequality 1 and Inequality 2 restrict the solutions to the first quadrant. Inequality 3 is the half-plane that lies on and below the line \(4x + 3y = 24\). The graph of the system of inequalities is the triangular region shown below.
GOAL 2 USING SYSTEMS OF INEQUALITIES IN REAL LIFE

You can use a system of linear inequalities to describe a real-life situation, as shown in the following example.

EXAMPLE 3 Writing and Using a System of Inequalities

HEART RATE A person’s theoretical maximum heart rate is \( 220 - x \) where \( x \) is the person’s age in years \((20 \leq x \leq 65)\). When a person exercises, it is recommended that the person strive for a heart rate that is at least 70% of the maximum and at most 85% of the maximum.

a. You are making a poster for health class. Write and graph a system of linear inequalities that describes the information given above.

b. A 40-year-old person has a heart rate of 150 (heartbeats per minute) when exercising. Is the person’s heart rate in the target zone?

SOLUTION

a. Let \( y \) represent the person’s heart rate. From the given information, you can write the following four inequalities.

\[
\begin{align*}
  x &\geq 20 & \text{Person’s age must be at least 20.} \\
  x &\leq 65 & \text{Person’s age can be at most 65.} \\
  y &\geq 0.7(220 - x) & \text{Target rate is at least 70% of maximum rate.} \\
  y &\leq 0.85(220 - x) & \text{Target rate is at most 85% of maximum rate.}
\end{align*}
\]

The graph of the system is shown below.

b. From the graph you can see that the target zone for a 40-year-old person is between 126 and 153, inclusive. That is,

\[
126 \leq y \leq 153.
\]

A 40-year-old person who has a heart rate of 150 is within the target zone.
**GUIDED PRACTICE**

**Vocabulary Check ✓**
1. What must be true in order for an ordered pair to be a solution of a system of linear inequalities?

**Concept Check ✓**
2. Look back at Example 1 on page 157. Explain why the ordered pair \((-1, -5)\) is not a solution of the system.

3. **ERROR ANALYSIS** Explain what is wrong with the graph of the following system of inequalities.

\[
\begin{align*}
y &\leq 3 \\
x + y &\geq 5
\end{align*}
\]

**Skill Check ✓**
Tell whether the ordered pair is a solution of the following system.

\[
\begin{align*}
x &\geq -1 \\
y &> 2x + 2
\end{align*}
\]

4. \((-1, 2)\)  
5. \((0, 0)\)  
6. \((1, 4)\)  
7. \((2, 7)\)

Graph the system of linear inequalities.

8. \[x \geq -1 \quad y > 2x + 2\]
9. \[x + y \leq 3 \quad y > 1\]
10. \[x > 0 \quad y \leq x - 5\]

11. **FLIGHT ATTENDANTS** To be a flight attendant, you must be at least 18 years old and at most 55 years old, and you must be between 60 and 74 inches tall, inclusive. Let \(x\) represent a person’s age (in years) and let \(y\) represent a person’s height (in inches). Write and graph a system of linear inequalities showing the possible ages and heights for flight attendants.

**PRACTICE AND APPLICATIONS**

**CHECKING A SOLUTION** Tell whether the ordered pair is a solution of the system.

12. \((25, -5)\)
13. \((2, 3)\)
14. \((2, 6)\)

**FINDING A SOLUTION** Give an ordered pair that is a solution of the system.

15. \[x - y \geq 3 \quad y < 15\]
16. \[x + y < 6 \quad x \geq -2\]
17. \[4x > y \quad x \leq 12\]
18. \[x \geq -7 \quad y < 10 \quad x < y\]
19. \[y > -5 \quad x > 3 \quad 2x + y < 13\]
20. \[y \geq -x \quad y \geq 0 \quad x < 0\]
**MATCHING SYSTEMS AND GRAPHS** Match the system of linear inequalities with its graph.

21. \(y \leq 4\) \(x > -2\)
22. \(y > -4\) \(x > -2\)
23. \(y > x\) \(x > -3\) \(y \geq 0\)
24. \(y > x\) \(y > -3\) \(x \leq 0\)
25. \(x \leq 3\) \(y > 1\) \(y \geq -x + 1\)
26. \(y > -1\) \(x \geq -1\) \(y \geq -x + 1\)

![Graphs A, B, C, D, E, F](image)

**SYSTEMS OF TWO INEQUALITIES** Graph the system of linear inequalities.

27. \(x < 5\) \(x > -4\)
28. \(y > -2\) \(y \leq 1\)
29. \(x \geq 0\) \(x + y < 11\)
30. \(x + y \geq -2\) \(-5x + y < -3\)
31. \(y \geq -4\) \(y < -2x + 10\)
32. \(y > 2x - 7\) \(4x + 4y < -12\)
33. \(y < x + 4\) \(y \geq -2x + 1\)
34. \(x + y > -8\) \(x + y \leq 6\)
35. \(y > -3x\) \(x \leq 5y\)
36. \(x - y > 7\) \(2x + y < 8\)
37. \(7x + y > 0\) \(3x - 2y \leq 5\)
38. \(-x < y\) \(x + 3y > 8\)

**SYSTEMS OF THREE OR MORE INEQUALITIES** Graph the system of linear inequalities.

39. \(y < 4\) \(x > -3\) \(y > x\)
40. \(y \geq 1\) \(x \leq 6\) \(y < 2x - 5\)
41. \(2x - 3y > -6\) \(5x - 3y < 3\) \(x + 3y > -3\)
42. \(x - 4y > 0\) \(x + y \leq 1\) \(x + 3y > -1\)
43. \(2x + 1 \geq y\) \(x < 5\) \(y < x + 2\)
44. \(5x - 3y \leq 4\) \(x + y < 8\) \(y > 3\)
45. \(x \geq y - 2\) \(x + y > 1\) \(x < 10\)
46. \(y \geq 0\) \(x - 4y < 2\) \(y < x\)
47. \(x - y \geq 0\) \(y < 2x\) \(5x + 6y \geq 1\)
48. \(y \geq 0\) \(x \leq 9\) \(x + y < 15\) \(y < x\)
49. \(x + y \leq 4\) \(x + y \geq -1\) \(x - y \geq -2\) \(x - y \leq 2\)
50. \(y < 5\) \(y > -6\) \(2x + y \geq -1\) \(y \leq x + 3\)
51. **POOL CHEMICALS** You are a lifeguard at a community pool, and you are in charge of maintaining the proper pH (amount of acidity) and chlorine levels. The water test-kit says that the pH level should be between 7.4 and 7.6 pH units and the chlorine level should be between 1.0 and 1.5 PPM (parts per million). Let \( p \) be the pH level and let \( c \) be the chlorine level (in PPM). Write and graph a system of inequalities for the pH and chlorine levels the water should have.

**HEALTH** In Exercises 52–54, use the following information. For a healthy person who is 4 feet 10 inches tall, the recommended lower weight limit is about 91 pounds and increases by about 3.7 pounds for each additional inch of height. The recommended upper weight limit is about 119 pounds and increases by about 4.9 pounds for each additional inch of height.

**Focus on People**

Ronny Weller of Germany set the world records for the snatch lift and combined lift in 1998. His records for these lifts are listed in the table for Ex. 57. Weller has won 3 Olympic medals: one gold, one silver, and one bronze.

**Shoe Sale** In Exercises 55 and 56, use the shoe store ad shown below.

52. Let \( x \) be the number of inches by which a person’s height exceeds 4 feet 10 inches and let \( y \) be the person’s weight in pounds. Write a system of inequalities describing the possible values of \( x \) and \( y \) for a healthy person.

53. Use a graphing calculator to graph the system of inequalities from Exercise 52.

54. What is the recommended weight range for someone 6 feet tall?

55. Let \( x \) be the regular footwear price and \( y \) be the discount price. Write a system of inequalities for the regular footwear prices and possible sale prices.

56. Graph the system you wrote in Exercise 55. Use your graph to estimate the range of possible sale prices for shoes that are regularly priced at $65.

57. **WEIGHTLIFTING RECORDS** The men’s world weightlifting records for the 105-kg-and-over weight category are shown in the table. The combined lift is the sum of the snatch lift and the clean and jerk lift. Let \( s \) be the weight lifted in the snatch and let \( j \) be the weight lifted in the clean and jerk. Write and graph a system of inequalities to describe the weights you could lift to break the records for both the snatch and combined lifts, but not the clean and jerk lift.

58. **Biology Connection** Each day, an average adult moose can process about 32 kilograms of terrestrial vegetation (twigs and leaves) and aquatic vegetation. From this food, it needs to obtain about 1.9 grams of sodium and 11,000 Calories of energy. Aquatic vegetation has about 0.15 gram of sodium per kilogram and about 193 Calories of energy per kilogram, while terrestrial vegetation has minimal sodium and about four times more energy than aquatic vegetation. Write and graph a system of inequalities describing the amounts \( t \) and \( a \) of terrestrial and aquatic vegetation, respectively, for the daily diet of an average adult moose.

**Data Update** of International Weightlifting Federation data at www.mcdougallittell.com

---

**Table 3.3.1**

<table>
<thead>
<tr>
<th>Men's +105 kg World Weightlifting Records</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snatch</td>
</tr>
<tr>
<td>205.5 kg</td>
</tr>
</tbody>
</table>

**Source:** Diet by Numbers
59. **CRITICAL THINKING** Write a system of three linear inequalities that has no solution. Graph the system to show that it has no solution.

60. **MULTIPLE CHOICE** Which system of inequalities is graphed at the right?

\[ \begin{align*}
\text{A} & : x + y > -5 \\
\text{B} & : x + y > -5 \\
\text{C} & : x + y > -5 \\
\text{D} & : x + y > -5
\end{align*} \]

-2x + y ≥ 3
-2x + y < 3
-2x + y ≤ 3
-2x + y > 3

61. **MULTIPLE CHOICE** Which ordered pair is **not** a solution of the following system of inequalities?

\[ \begin{align*}
3x + 2y & \geq -2 \\
x - y & < 3
\end{align*} \]

- \( \text{A} \) \((0, 0)\)
- \( \text{B} \) \((-1, 2)\)
- \( \text{C} \) \((4, 1)\)
- \( \text{D} \) \((2, 2)\)

**Challenge**

**WRITING A SYSTEM** Write a system of linear inequalities for the region.

62. 63. 64.

65. **VISUAL THINKING** Write a system of linear inequalities whose graph is a pentagon and its interior.

**MIXED REVIEW**

**EVALUATING EXPRESSIONS** Evaluate the expression for the given values of \( x \) and \( y \). (Review 1.2 for 3.4)

- 66. \( 2x + 7y \) when \( x = 5 \) and \( y = -3 \)
- 67. \( -4x - 3y \) when \( x = -6 \) and \( y = -1 \)
- 68. \( 10x - 3y \) when \( x = -4 \) and \( y = 2 \)
- 69. \( -y + 8x \) when \( y = -3 \) and \( x = -2 \)

**DETERMINING CORRELATION** Tell whether \( x \) and \( y \) have a **positive correlation**, a **negative correlation**, or **relatively no correlation**. (Review 2.5)

- 70. 
- 71. 
- 72. 

**CHOOSING A METHOD** Solve the system using any algebraic method. (Review 3.2)

- 73. \( 13x + 5y = 2 \)
- 74. \( -2x + 7y = 10 \)
- 75. \( 5x + 6y = -12 \)
- 76. \( -7x + 5y = 0 \)
- 77. \( -4x - 10y = 12 \)
- 78. \( 6x - 8y = -18 \)
- 79. \( x - 4y = 10 \)
- 80. \( x - 3y = -3 \)
- 81. \( 10x + 12y = 24 \)
- 82. \( 14x - 8y = 2 \)
- 83. \( x + 5y = 2 \)
- 84. \( -3x + 4y = 9 \)
Linear Programming

**Goal 1: Using Linear Programming**

Many real-life problems involve a process called **optimization**, which means finding the maximum or minimum value of some quantity. In this lesson, you will study one type of optimization process called **linear programming**.

**Linear programming** is the process of optimizing a linear **objective function** subject to a system of linear inequalities called **constraints**. The graph of the system of constraints is called the **feasible region**.

**Activity: Investigating Linear Programming**

1. Evaluate the objective function \( C = 2x + 4y \) for each labeled point in the feasible region at the right.
2. At which labeled point does the maximum value of \( C \) occur? At which labeled point does the minimum value of \( C \) occur?
3. What are the maximum and minimum values of \( C \) on the entire feasible region? Try other points in the region to see if you can find values of \( C \) that are greater or lesser than those you found in Step 2.

Constraints:
- \( x \geq 0 \)
- \( y \geq 0 \)
- \( -x + 3y \leq 15 \)
- \( 2x + y \leq 12 \)

In the activity, you may have discovered that the optimal values of the objective function occurred at vertices of the feasible region.

**Optimal Solution of a Linear Programming Problem**

If an objective function has a maximum or a minimum value, then it must occur at a vertex of the feasible region. Moreover, the objective function will have both a maximum and a minimum value if the feasible region is bounded.
EXAMPLE 1 \textbf{Solving a Linear Programming Problem}

Find the minimum value and the maximum value of
\[ C = 3x + 4y \]
\textbf{Objective function}
subject to the following constraints.
\[
\begin{align*}
  x &\geq 0 \\
  y &\geq 0 \\
  x + y &\leq 8
\end{align*}
\]
\textbf{Constraints}

\textbf{SOLUTION}

The feasible region determined by the constraints is shown. The three vertices are \((0, 0), (8, 0),\) and \((0, 8)\). To find the minimum and maximum values of \(C\), evaluate \(C = 3x + 4y\) at each of the three vertices.

At \((0, 0)\):
\[ C = 3(0) + 4(0) = 0 \]
minimum

At \((8, 0)\):
\[ C = 3(8) + 4(0) = 24 \]
maximum

At \((0, 8)\):
\[ C = 3(0) + 4(8) = 32 \]

The minimum value of \(C\) is 0. It occurs when \(x = 0\) and \(y = 0\). The maximum value of \(C\) is 32. It occurs when \(x = 0\) and \(y = 8\).

EXAMPLE 2 \textbf{A Region that is Unbounded}

Find the minimum value and the maximum value of
\[ C = 5x + 6y \]
\textbf{Objective function}
subject to the following constraints.
\[
\begin{align*}
  x &\geq 0 \\
  y &\geq 0 \\
  x + y &\geq 5 \\
  3x + 4y &\geq 18
\end{align*}
\]
\textbf{Constraints}

\textbf{SOLUTION}

The feasible region determined by the constraints is shown. The three vertices are \((0, 5), (2, 3),\) and \((6, 0)\). First evaluate \(C = 5x + 6y\) at each of the vertices.

At \((0, 5)\):
\[ C = 5(0) + 6(5) = 30 \]

At \((2, 3)\):
\[ C = 5(2) + 6(3) = 28 \]

At \((6, 0)\):
\[ C = 5(6) + 6(0) = 30 \]

If you evaluate several other points in the feasible region, you will see that as the points get farther from the origin, the value of the objective function increases without bound. Therefore, the objective function has no maximum value. Since the value of the objective function is always at least 28, the minimum value is 28.
**GOAL 2** LINEAR PROGRAMMING IN REAL LIFE

**EXAMPLE 3 Using Linear Programming to Find the Maximum Profit**

**BICYCLE MANUFACTURING** Two manufacturing plants make the same kind of bicycle. The table gives the hours of general labor, machine time, and technical labor required to make one bicycle in each plant. For the two plants combined, the manufacturer can afford to use up to 4000 hours of general labor, up to 1500 hours of machine time, and up to 2300 hours of technical labor per week. Plant A earns a profit of $60 per bicycle and Plant B earns a profit of $50 per bicycle. How many bicycles per week should the manufacturer make in each plant to maximize profit?

<table>
<thead>
<tr>
<th>Resource</th>
<th>Hours per bicycle in Plant A</th>
<th>Hours per bicycle in Plant B</th>
</tr>
</thead>
<tbody>
<tr>
<td>General labor</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Machine time</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Technical labor</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

**Solution**

**Write** an objective function. Let \(a\) and \(b\) represent the number of bicycles made in Plant A and Plant B, respectively. Because the manufacturer wants to maximize the profit \(P\), the objective function is:

\[ P = 60a + 50b \]

**Write** the constraints in terms of \(a\) and \(b\). The constraints are given below and the feasible region determined by the constraints is shown at the right.

1. \(10a + b \leq 4000\) 
   *General labor: up to 4000 hours*
2. \(a + 3b \leq 1500\) 
   *Machine time: up to 1500 hours*
3. \(5a + 2b \leq 2300\) 
   *Technical labor: up to 2300 hours*
4. \(a \geq 0\) 
   *Cannot produce a negative amount*
5. \(b \geq 0\) 
   *Cannot produce a negative amount*

**Calculate** the profit at each vertex of the feasible region.

- At \((0, 500)\): \[ P = 60(0) + 50(500) = 25,000 \]
- At \((300, 400)\): \[ P = 60(300) + 50(400) = 38,000 \]
- At \((380, 200)\): \[ P = 60(380) + 50(200) = 32,800 \]
- At \((400, 0)\): \[ P = 60(400) + 50(0) = 24,000 \]
- At \((0, 0)\): \[ P = 60(0) + 50(0) = 0 \]

The maximum profit is obtained by making 300 bicycles in Plant A and 400 bicycles in Plant B.
**GUIDED PRACTICE**

1. Define linear programming.
2. How is the objective function used in a linear programming problem? How is the system of constraints used?
3. In a linear programming problem, which ordered pairs should be tested to find a minimum or maximum value?

**Skill Check ✓**

In Exercises 4 and 5, use the feasible region at the right.

4. What are the vertices of the feasible region?
5. What are the minimum and maximum values of the objective function \( C = 5x + 7y \)?

Find the minimum and maximum values of the objective function subject to the given constraints.

6. **Objective function:** \( C = x + y \); **Constraints:** \( y \leq 5, y \geq 0, y - 2x \geq 0 \)
7. **Objective function:** \( C = 2x - y \); **Constraints:** \( x \geq 0, x + y \leq 20, y \geq 3 \)
8. Planning a Fundraiser Your club plans to raise money by selling two sizes of fruit baskets. The plan is to buy small baskets for $10 and sell them for $16 and to buy large baskets for $15 and sell them for $25. The club president estimates that you will not sell more than 100 baskets. Your club can afford to spend up to $1200 to buy the baskets. Find the number of small and large fruit baskets you should buy in order to maximize profit.

**PRACTICE AND APPLICATIONS**

**Checking Vertices** Find the minimum and maximum values of the objective function for the given feasible region.

9. \( C = x - y \)
10. \( C = 2x + 5y \)
11. \( C = 4x + 2y \)

**Finding Values** In Exercises 12–20, find the minimum and maximum values of the objective function subject to the given constraints.

12. **Objective function:** \( C = 2x + 3y \)  
   **Constraints:** 
   \( x \geq 0 \) 
   \( y \geq 0 \) 
   \( x + y \leq 9 \)
13. **Objective function:** \( C = x + 4y \)  
   **Constraints:** 
   \( x \geq 2 \) 
   \( x \leq 5 \) 
   \( y \geq 1 \) 
   \( y \leq 6 \)
14. **Objective function:** \( C = 2x + y \)  
   **Constraints:** 
   \( x \geq -5 \) 
   \( x \leq 0 \) 
   \( y \geq -2 \) 
   \( y \leq 2 \)
15. Objective function:  
   \( C = 10x + 7y \)  
   Constraints:  
   \( 0 \leq x \leq 60 \)  
   \( 0 \leq y \leq 45 \)  
   \( 5x + 6y \leq 420 \)

16. Objective function:  
   \( C = -2x + y \)  
   Constraints:  
   \( x \geq 0 \)  
   \( y \geq 0 \)  
   \( x + y \geq 7 \)  
   \( 5x + 2y \geq 20 \)

17. Objective function:  
   \( C = 4x + 6y \)  
   Constraints:  
   \( -x + y \leq 11 \)  
   \( x + y \leq 27 \)  
   \( 2x + 5y \leq 90 \)

18. Objective function:  
   \( C = 5x + 4y \)  
   Constraints:  
   \( x \geq 0 \)  
   \( y \geq 0 \)  
   \( x + y \leq 14 \)  
   \( 5x + y \leq 50 \)

19. Objective function:  
   \( C = 4x + 3y \)  
   Constraints:  
   \( x \geq 0 \)  
   \( 2x + 3y \geq 6 \)  
   \( 3x - 2y \leq 9 \)  
   \( x + 5y \leq 20 \)

20. Objective function:  
   \( C = 10x + 3y \)  
   Constraints:  
   \( x \geq 0 \)  
   \( y \geq 0 \)  
   \( -x + y \geq 0 \)  
   \( 2x + y \leq 4 \)  
   \( 2x + y \leq 13 \)

21. **Juice Blends**  
   A juice company makes two kinds of juice: Orangeade and Berry-fruity. One gallon of Orangeade is made by mixing 2.5 quarts of orange juice and 1.5 quarts of raspberry juice, while one gallon of Berry-fruity is made by mixing 3 quarts of raspberry juice and 1 quart of orange juice. A profit of $.50 is made on every gallon of Orangeade sold, and a profit of $.40 is made on every gallon of Berry-fruity sold. If the company has 150 gallons of raspberry juice and 125 gallons of orange juice on hand, how many gallons of each type of juice should be made to maximize profit?

22. **File Cabinets**  
   An office manager is purchasing file cabinets and wants to maximize storage space. The office has 60 square feet of floor space for the cabinets and $600 in the budget to purchase them. Cabinet A requires 3 square feet of floor space, has a storage capacity of 12 cubic feet, and costs $75. Cabinet B requires 6 square feet of floor space, has a storage capacity of 18 cubic feet, and costs $50. How many of each cabinet should the office manager buy?

23. **Home Canning**  
   You have 180 tomatoes and 15 onions left over from your garden. You want to use these to make jars of tomato sauce and jars of salsa to sell at a farm stand. A jar of tomato sauce requires 10 tomatoes and 1 onion, and a jar of salsa requires 5 tomatoes and 1 onion. You’ll make a profit of $2 on every jar of tomato sauce sold and a profit of $1.50 on every jar of salsa sold. The farm stand wants at least three times as many jars of tomato sauce as jars of salsa. How many jars of each should you make to maximize profit?

24. **Nutrition**  
   You are planning a dinner of pinto beans and brown rice. You want to consume at least 2100 Calories and 44 grams of protein per day, but no more than 2400 milligrams of sodium and 73 grams of fat. So far today, you have consumed 1600 Calories, 24 grams of protein, 2370 milligrams of sodium, and 65 grams of fat. Pinto beans cost $.57 per cup and brown rice costs $.78 per cup. How many cups of pinto beans and brown rice should you make to minimize cost while satisfying your nutritional requirements?

<table>
<thead>
<tr>
<th>Content</th>
<th>1 cup pinto beans</th>
<th>1 cup brown rice (with salt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calories</td>
<td>265</td>
<td>230</td>
</tr>
<tr>
<td>Protein (g)</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>Sodium (mg)</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Fat (g)</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

3.4 Linear Programming
25. **MULTIPLE CHOICE** Given the feasible region shown, what is the maximum value of the objective function
   \[ C = 2x + 6y \]?

- A 0
- B 60
- C 200
- D 276
- E 326

26. **MULTIPLE CHOICE** Given the constraints \( y \geq 0, y \leq x + 8, \) and \( y \geq 2x + 8, \)
   what is the minimum value of the objective function \( C = -2x - y \)?

- A -8
- B 16
- C -16
- D 8

27. **CONSECUTIVE VERTICES** Find the value of the objective function at each vertex of the feasible region and at two points on each line segment connecting two vertices. What can you conclude?

   a. Objective function:
   \[ C = 2x + 2y \]
   Constraints:
   \[ y \leq 4 \]
   \[ x \leq 5 \]
   \[ x + y \leq 6 \]

   b. Objective function:
   \[ C = 5x - y \]
   Constraints:
   \[ y \geq -1 \]
   \[ x \leq 3 \]
   \[ -5x + y \leq 4 \]

---

**MIXED REVIEW**

**GRAPHING EQUATIONS** Graph the equation. Label any intercepts.
(Review 2.3 for 3.5)

28. \( x - y = 10 \)
29. \( 3x + 4y = -12 \)
30. \( y = -3x + 2 \)
31. \( 5x - 15y = 15 \)
32. \( y = -\frac{3}{4}x + 2 \)
33. \( y = -\frac{1}{2}x + 7 \)

**EVALUATING FUNCTIONS** Evaluate the function for the given value of \( x \).
(Review 2.7)

34. \( f(0) \)
35. \( f(-2) \)
36. \( f(-10) \)
37. \( f(-1) \)
38. \( g(1) \)
39. \( g(-5) \)
40. \( g(-1) \)
41. \( g(7) \)

**GRAPHING SYSTEMS OF INEQUALITIES** Graph the system of linear inequalities. (Review 3.3)

42. \( x > 2 \)
   \( y < 6 \)

43. \( x + y \leq 5 \)
   \( y > 0 \)

44. \( x < -1 \)
   \( x - y \geq 4 \)

45. \( y < 5 \)
   \( x \geq -1 \)
   \( y \geq 1 \)

46. \( -x + y > 2 \)
   \( y > 0 \)
   \( 2x + y \leq 3 \)

47. \( x + y \leq 6 \)
   \( -\frac{1}{2}x + y \leq 3 \)
   \( y \leq 3 \)

48. **AMUSEMENT CENTER** You have 30 tokens for playing video games and pinball. It costs 3 tokens to play a video game and 2 tokens to play pinball. You want to play an equal number of video games and pinball games. Use an algebraic model to find how many games of each you can play. (Review 1.5)
Graph the system of linear inequalities. (Lesson 3.3)

1. \( y > -2 \)
   \( x ≥ -4 \)
   \( y ≤ -x + 1 \)

2. \( y > -5 \)
   \( x ≤ 2 \)
   \( y ≤ x + 2 \)

3. \( x ≤ 3 \)
   \( y < 2 \)
   \( y > -x + 1 \)

Find the minimum and maximum values of the objective function \( C = 5x + 2y \) subject to the given constraints. (Lesson 3.4)

4. Constraints:
   \( x ≤ -2 \)
   \( x ≥ 0 \)
   \( y ≥ 1 \)
   \( y ≤ 6 \)

5. Constraints:
   \( x ≥ 0 \)
   \( y ≥ 2 \)
   \( 2x + y ≤ 10 \)
   \( x - 3y ≥ -3 \)

6. Constraints:
   \( x ≥ 0 \)
   \( y ≥ 0 \)
   \( y ≤ 8 \)
   \( x + y ≤ 14 \)

7. **Maximum Income** You are stenciling wooden boxes to sell at a fair. It takes you 2 hours to stencil a small box and 3 hours to stencil a large box. You make a profit of $10 for a small box and $20 for a large box. If you have no more than 30 hours available to stencil and want at least 12 boxes to sell, how many of each size box should you stencil to maximize your profit? (Lesson 3.4)

---

**Math & History**

**Linear Programming in World War II**

**THEN**

**During World War II**, the need for efficient transportation of supplies inspired mathematician George Dantzig to develop linear programming.

The LST was a ship used during World War II that carried 3 ton trucks and 25 ton tanks. The upper deck could carry 27 trucks, but no tanks. The tank deck could carry 500 tons, but no more than 33 trucks.

1. What is the maximum number of tanks that an LST could hold?
2. What is the maximum number of trucks that an LST could hold?
3. Suppose an LST was to be loaded with as many tanks and trucks as possible, and at least three times as many trucks as tanks. What is the maximum number of tanks and trucks that could be loaded?

**NOW**

**In 1984** mathematician Narendra Karmarkar developed a new time-saving linear programming method. Today his method is used by industries that deal with allocation of resources, such as telephone companies, airlines, and manufacturers.

- **1947** George Dantzig develops simplex method.
- **1975** L.G. Khachyan develops ellipsoid method.
- **1979** L.V. Kantorovich and T.C. Koopmans receive Nobel Prize for their linear programming work.
- **1984** N. Karmarkar devises a polynomial-time algorithm.
Graphing Linear Equations in Three Variables

**GOAL 1** Graphing in Three Dimensions

Solutions of equations in three variables can be pictured with a three-dimensional coordinate system. To construct such a system, begin with the $xy$-coordinate plane in a horizontal position. Then draw the $z$-axis as a vertical line through the origin.

In much the same way that points in a two-dimensional coordinate system are represented by ordered pairs, each point in space can be represented by an ordered triple $(x, y, z)$.

Drawing the point represented by an ordered triple is called **plotting** the point.

The three axes, taken two at a time, determine three coordinate planes that divide space into eight octants. The first octant is the one for which all three coordinates are positive.

**EXAMPLE 1** Plotting Points in Three Dimensions

Plot the ordered triple in a three-dimensional coordinate system.

- **a.** $(−5, 3, 4)$
- **b.** $(3, −4, −2)$

**Solution**

- **a.** To plot $(−5, 3, 4)$, it helps to first find the point $(-5, 3)$ in the $xy$-plane. The point $(-5, 3, 4)$ lies four units above.
- **b.** To plot $(3, −4, −2)$, find the point $(3, −4)$ in the $xy$-plane. The point $(3, −4, −2)$ lies two units below.
A **linear equation in three variables** \( x, y, \text{ and } z \) is an equation of the form

\[ ax + by + cz = d \]

where \( a, b, \text{ and } c \) are not all zero. An ordered triple \((x, y, z)\) is a *solution* of this equation if the equation is true when the values of \( x, y, \text{ and } z \) are substituted into the equation. The *graph* of an equation in three variables is the graph of all its solutions. The graph of a linear equation in three variables is a plane.

### EXAMPLE 2  
**Graphing a Linear Equation in Three Variables**

Sketch the graph of \( 3x + 2y + 4z = 12 \).

**SOLUTION**

Begin by finding the points at which the graph intersects the axes. Let \( x = 0 \) and \( y = 0 \), and solve for \( z \) to get \( z = 3 \). This tells you that the \( z \)-intercept is 3, so plot the point \((0, 0, 3)\). In a similar way, you can find that the \( x \)-intercept is 4 and the \( y \)-intercept is 6. After plotting \((0, 0, 3), (4, 0, 0), \text{ and } (0, 6, 0)\), you can connect these points with lines to form the triangular region of the plane that lies in the first octant.

A linear equation in \( x, y, \text{ and } z \) can be written as a **function of two variables**. To do this, solve the equation for \( z \). Then replace \( z \) with \( f(x, y) \).

### EXAMPLE 3  
**Evaluating a Function of Two Variables**

a. Write the linear equation \( 3x + 2y + 4z = 12 \) as a function of \( x \) and \( y \).

b. Evaluate the function when \( x = 1 \) and \( y = 3 \). Interpret the result geometrically.

**SOLUTION**

a. \( 3x + 2y + 4z = 12 \)  
   \[ 4z = 12 - 3x - 2y \]  
   \[ z = \frac{1}{4}(12 - 3x - 2y) \]  
   \[ f(x, y) = \frac{1}{4}(12 - 3x - 2y) \]  
   Replace \( z \) with \( f(x, y) \).

b. \( f(1, 3) = \frac{1}{4}(12 - 3(1) - 2(3)) = \frac{3}{4} \). This tells you that the graph of \( f \) contains the point \( \left(1, 3, \frac{3}{4}\right)\).
GOAL 2

USING FUNCTIONS OF TWO VARIABLES IN REAL LIFE

EXAMPLE 4  Modeling a Real-Life Situation

LANDSCAPING  You are planting a lawn and decide to use a mixture of two types of grass seed: bluegrass and rye. The bluegrass costs $2 per pound and the rye costs $1.50 per pound. To spread the seed you buy a spreader that costs $35.

a. Write a model for the total amount you will spend as a function of the number of pounds of bluegrass and rye.

b. Evaluate the model for several different amounts of bluegrass and rye, and organize your results in a table.

SOLUTION

a. Your total cost involves two variable costs (for the two types of seed) and one fixed cost (for the spreader).

\[
C = 2x + 1.5y + 35
\]

b. To evaluate the function of two variables, substitute values of \(x\) and \(y\) into the function. For instance, when \(x = 10\) and \(y = 20\), the total cost is:

\[
C = 2(10) + 1.5(20) + 35 = 20 + 30 + 35 = 85
\]

The table shows the total cost for several different values of \(x\) and \(y\).
Graphing Linear Equations in Three Variables

1. Write the general form of a linear equation in three variables. How is the solution of such an equation represented?

2. **LOGICAL REASONING** Tell whether this statement is true or false: The graph of a linear equation in three variables consists of three different lines.

3. How are octants and quadrants similar?

4. Describe how you would graph a linear equation in three variables.

5. Draw a three-dimensional coordinate system and plot the ordered triple (2, -4, -6).

6. Write the coordinates of the vertices A, B, C, and D of the rectangular prism shown, given that one vertex is the point (2, 3, 4).

7. Sketch the graph of the equation. Label the points where the graph crosses the x-, y-, and z-axes.

   7.  $8x + 4y + 2z = 16$
   8.  $2x + 4y + 5z = 20$
   9.  $3x + 3y + 7z = 21$
   10. $10x + 2y + 5z = 10$
   11. $9x + 3y + 3z = 27$
   12. $4x + y + 2z = 8$

8. Write the linear equation as a function of $x$ and $y$. Then evaluate the function for the given values.

   13. $6x + 6y + 3z = 9$, $f(1, 2)$
   14. $-2x - y + z = 7$, $f(-3, 2)$
   15. $8x + 2y + 4z = -16$, $f(5, 6)$
   16. $5x - 10y - 5z = 15$, $f(2, 2)$

17. **TRAIL MIX** You are making bags of a trail mix called GORP (Good Old Raisins and Peanuts). The raisins cost $2.25 per pound and the peanuts cost $2.95 per pound. The package of bags for the trail mix costs $2.65. Write a model for the total cost as a function of the number of pounds of raisins and peanuts you buy. Evaluate the model for 5 lb of raisins and 8 lb of peanuts.

**Plotting Points** Plot the ordered triple in a three-dimensional coordinate system.

18. (2, 4, 0)  19. (4, -1, -6)  20. (5, -2, -2)  21. (0, 6, -3)
   22. (3, 4, -2)  23. (-2, 1, 1)  24. (5, -1, 5)  25. (-3, 2, -7)

**Sketching Graphs** Sketch the graph of the equation. Label the points where the graph crosses the x-, y-, and z-axes.

26. $x + y + z = 7$
27. $5x + 4y + 2z = 20$
28. $x + 6y + 4z = 12$
29. $12x + 3y + 8z = 24$
30. $2x + 18y + 3z = 36$
31. $7x + 9y + 21z = 63$
32. $7x + 7y + 2z = 14$
33. $6x + 4y + 3z = 10$
34. $3x + 5y + 3z = 15$
35. $\frac{1}{2}x + 4y - 3z = 8$
36. $5x + y + 2z = -4$
37. $-2x + 9y + 3z = 18$
EVALUATING FUNCTIONS  Write the linear equation as a function of \( x \) and \( y \). Then evaluate the function for the given values.

38. \( 6x + 2y + 3z = 18, f(2, 1) \)
39. \( -2x - 5y + 5z = 15, f\left(\frac{3}{2}, -2\right) \)
40. \( x + 6y + z = 10, f(-4, -1) \)
41. \( 3x - \frac{3}{4}y + \frac{5}{2}z = 9, f(-3, 16) \)
42. \( -x - 2y - 7z = 14, f(-5, -10) \)
43. \( 10x + 15y + 60z = 12, f\left(-\frac{3}{4}, \frac{4}{5}\right) \)
44. \( x - 5y - z = 14, f(3, 6) \)
45. \( -x + 6y - 9z = 12, f\left(-\frac{1}{2}, 12\right) \)

46. GEOMETRY CONNECTION  Use the given point \((4, 7, 2)\) to find the volume of the rectangular prism.

47. GEOMETRY CONNECTION  Use the given point \((5, 6, -2)\) to find the volume of the rectangular prism.

48. HOME AQUARIUM  You want to buy an aquarium and stock it with goldfish and angelfish. The pet store sells goldfish for \$0.40 each and angelfish for \$4 each. The aquarium starter kit costs \$65. Write a model for the amount you will spend as a function of the number of goldfish and angelfish you buy. Make a table that shows the total cost for several different numbers of goldfish and angelfish.

49. POTTERY  A craft store has paint-your-own pottery sessions available. You pick out a piece of pottery that ranges in price from \$8 to \$50 and pick out paint colors for \$1.50 per color. The craft store charges a base fee of \$16 for sitting time, brushes, glaze, and kiln time. Write a model for the total cost of making a piece of pottery as a function of the price of the pottery and the number of paint colors you use. Make a table that shows the total cost for several different pieces of pottery and numbers of paint colors.

50. FLOWER ARRANGEMENT  You are buying tulips, carnations, and a glass vase to make a flower arrangement. The flower shop sells tulips for \$0.70 each and carnations for \$0.30 each. The glass vase costs \$12. Write a model for the total cost of the flower arrangement as a function of the number of tulips and carnations you use. Make a table that shows the total cost for several different numbers of tulips and carnations.

51. TRANSPORTATION  Every month you buy a local bus pass for \$20 that is worth \$0.60 toward the fare for the local bus, the express bus, or the subway. The local bus costs \$0.60, the express bus costs \$1.50, and the subway costs \$0.85. Write a model for the total cost of transportation in a month as a function of the number of times you take the express bus and the number of times you take the subway. Evaluate the model for 8 express bus rides and 10 subway rides. Make a table that shows the total cost for several different numbers of rides.
52. **AFTER-SCHOOL JOBS** Several days after school you are a lifeguard at a community pool. On weekends you baby-sit to earn extra money. Lifeguarding pays $8 per hour and baby-sitting pays $6 per hour. You also get a weekly allowance of $10 for doing chores around the house. Write an equation for your total weekly earnings as a function of the number of hours you lifeguard and baby-sit. Make a table that shows several different amounts of weekly earnings.

53. **MULTI-STEP PROBLEM** You are deciding how many times to air a 60 second commercial on a radio station. The station charges $100 for a 60 second spot during off-peak listening hours and $350 for a 60 second spot during peak listening hours. The company you have hired to make your commercial charges $500.

   a. Write a model for the total amount that will be spent making and airing the commercial as a function of the number of times it is aired during off-peak and peak listening hours.

   b. Evaluate the model for several different numbers of off-peak airings and peak airings. Organize your results in a table.

   c. Writing Suppose your advertising budget is $4000. Using the table you made in part (b), can you air the commercial 8 times during off-peak hours and 8 times during peak hours? What combination of off-peak and peak airings would you recommend? Explain.

**WRITING EQUATIONS** Write an equation of the plane having the given \(x\), \(y\), and \(z\)-intercepts. Explain the method you used.

-  **54.** \(x\)-intercept: 4
  \(y\)-intercept: -2
  \(z\)-intercept: 4

-  **55.** \(x\)-intercept: \(\frac{3}{2}\)
  \(y\)-intercept: 12
  \(z\)-intercept: 6

-  **56.** \(x\)-intercept: 4
  \(y\)-intercept: -6
  \(z\)-intercept: -9

**MIXED REVIEW**

**SOLVING INEQUALITIES** Solve the inequality. Then graph the solution.

(R Review 1.6)

-  **57.** \(3 + x \leq 17\)
-  **58.** \(2x + 5 \geq 21\)
-  **59.** \(-x + 3 < 3x + 11\)
-  **60.** \(-13 < 6x - 1 < 11\)
-  **61.** \(24 \leq 2x - 12 \leq 30\)
-  **62.** \(-3 < 2x - 3 \leq 17\)

**TYPES OF LINES** Tell whether the lines are **parallel**, **perpendicular**, or **neither**.

(R Review 2.2)

-  **63.** Line 1: through (1, 7) and (-3, -5)
   Line 2: through (-6, 20) and (0, 2)
-  **64.** Line 1: through (4, -4) and (-16, 1)
   Line 2: through (1, 5) and (5, 21)
-  **65.** Line 1: through (-2, 1) and (0, 3)
   Line 2: through (2, 1) and (0, -1)
-  **66.** Line 1: through (0, 6) and (5, -2)
   Line 2: through (-1, -1) and (7, 4)

**HOME CARPENTRY** You have budgeted $48.50 to purchase red oak and poplar boards to make a bookcase. Each red oak board costs $3.95 and each poplar board costs $3.10. You need a total of 14 boards for the bookcase. Write and solve a system of equations to find the number of red oak boards and the number of poplar boards you should buy. (Review 3.1, 3.2 for 3.6)
Graphing Linear Equations in Three Variables

Some graphing calculators can be used to graph a linear equation in three variables. The instructions for graphing on a TI-92 are given below.

**EXAMPLE**

Use a graphing calculator (or a computer) to graph the equation $3x + 5y + 6z = 30$.

**SOLUTION**

1. Solve the equation for $z$.
   
   $3x + 5y + 6z = 30$
   
   $6z = 30 - 3x - 5y$
   
   $z = 5 - \frac{1}{2}x - \frac{5}{6}y$

2. Enter the equation in the $[Z=]$ editor.

3. Display the axes in box format and turn the labels on.

4. Set the window values as shown.

5. Graph the equation. You can use the Evaluate feature to evaluate $z$ for values of $x$ and $y$.

**EXERCISES**

Use a graphing calculator (or a computer) to graph the equation. Then evaluate $z$ for the given values of $x$ and $y$.

1. $4x + 18y + 3z = 54$; $x = 6$, $y = 4$
2. $3x + y + z = 24$; $x = 1.5$, $y = 19$
3. $x + 3y + 10z = 45$; $x = 20$, $y = 7$
4. $7x + 6y + 2z = 61$; $x = 4$, $y = 4$
5. $4x + 13y - 5z = 26$; $x = 14$, $y = 6$
6. $3x - 25y + 20z = 35$; $x = 5$, $y = 0$
Solving Systems of Linear Equations in Three Variables

**GOAL 1** **SOLVING A SYSTEM IN THREE VARIABLES**

In Lessons 3.1 and 3.2 you learned how to solve a system of two linear equations in two variables. In this lesson you will learn how to solve a **system of three linear equations** in three variables. Here is an example.

\[
\begin{align*}
    x + 2y - 3z &= -3 & \text{Equation 1} \\
    2x - 5y + 4z &= 13 & \text{Equation 2} \\
    5x + 4y - z &= 5 & \text{Equation 3}
\end{align*}
\]

A **solution** of such a system is an ordered triple \((x, y, z)\) that is a solution of all three equations. For instance, \((2, -1, 1)\) is a solution of the system above.

\[
\begin{align*}
    2 + 2(-1) - 3(1) &= 2 - 2 - 3 = -3 \checkmark \\
    2(2) - 5(-1) + 4(1) &= 4 + 5 + 4 = 13 \checkmark \\
    5(2) + 4(-1) - 1 &= 10 - 4 - 1 = 5 \checkmark
\end{align*}
\]

From Lesson 3.5 you know that the graph of a linear equation in three variables is a plane. Three planes in space can intersect in different ways.

If the planes intersect in a single point, as shown below, the system has exactly one solution.

If the planes intersect in a line, as shown below, the system has infinitely many solutions.

If the planes have no point of intersection, the system has no solution. In the example on the left, the planes intersect pairwise, but all three have no points in common. In the example on the right, the planes are parallel.
The linear combination method you learned in Lesson 3.2 can be extended to solve a system of linear equations in three variables.

**THE LINEAR COMBINATION METHOD (3-VARIABLE SYSTEMS)**

**STEP 1** Use the linear combination method to rewrite the linear system in three variables as a linear system in two variables.

**STEP 2** Solve the new linear system for both of its variables.

**STEP 3** Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

*Note:* If you obtain a false equation, such as $0 = 1$, in any of the steps, then the system has no solution. If you do not obtain a false solution, but obtain an identity, such as $0 = 0$, then the system has infinitely many solutions.

**EXAMPLE 1** *Using the Linear Combination Method*

Solve the system.

\[
\begin{align*}
3x + 2y + 4z &= 11 & \text{Equation 1} \\
2x - y + 3z &= 4 & \text{Equation 2} \\
5x - 3y + 5z &= -1 & \text{Equation 3}
\end{align*}
\]

**SOLUTION**

1. Eliminate one of the variables in two of the original equations.

\[
\begin{align*}
3x + 2y + 4z &= 11 & \text{Add 2 times the second} \\
4x - 2y + 6z &= 8 & \text{equation to the first.} \\
7x + 10z &= 19 & \text{New Equation 1} \\
5x - 3y + 5z &= -1 & \text{Add } -3 \text{ times the second} \\
-6x + 3y - 9z &= -12 & \text{equation to the third.} \\
-x - 4z &= -13 & \text{New Equation 2}
\end{align*}
\]

2. Solve the new system of linear equations in two variables.

\[
\begin{align*}
7x + 10z &= 19 & \text{New Equation 1} \\
-7x - 28z &= -91 & \text{Add 7 times new Equation 2.} \\
-18z &= -72 & \text{Solve for } z. \\
z &= 4 & \\
x &= -3 & \text{Substitute into new Equation 1 or 2 to find } x.
\end{align*}
\]

3. Substitute $x = -3$ and $z = 4$ into an original equation and solve for $y$.

\[
\begin{align*}
2x - y + 3z &= 4 & \text{Equation 2} \\
2(-3) - y + 3(4) &= 4 & \text{Substitute } -3 \text{ for } x \text{ and } 4 \text{ for } z. \\
y &= 2 & \text{Solve for } y.
\end{align*}
\]

The solution is $x = -3$, $y = 2$, and $z = 4$, or the ordered triple $(-3, 2, 4)$. Check this solution in each of the original equations.
**EXAMPLE 2  Solving a System with No Solution**

Solve the system.

\[
\begin{align*}
    x + y + z &= 2 & \text{Equation 1} \\
    3x + 3y + 3z &= 14 & \text{Equation 2} \\
    x - 2y + z &= 4 & \text{Equation 3}
\end{align*}
\]

**Solution**

When you multiply the first equation by \(-3\) and add the result to the second equation, you obtain a false equation.

\[
\begin{align*}
    -3x - 3y - 3z &= -6 & \text{Add } -3 \text{ times the first} \\
    3x + 3y + 3z &= 14 & \text{equation to the second.} \\
    0 &= 8 & \text{New Equation 1}
\end{align*}
\]

Because you obtained a false equation, you can conclude that the original system of equations has no solution.

**EXAMPLE 3  Solving a System with Many Solutions**

Solve the system.

\[
\begin{align*}
    x + y + z &= 2 & \text{Equation 1} \\
    x + y - z &= 2 & \text{Equation 2} \\
    2x + 2y + z &= 4 & \text{Equation 3}
\end{align*}
\]

**Solution**

*Rewrite* the linear system in three variables as a linear system in two variables.

\[
\begin{align*}
    x + y + z &= 2 & \text{Add the first equation} \\
    x + y - z &= 2 & \text{to the second.} \\
    2x + 2y &= 4 & \text{New Equation 1} \\
    x + y - z &= 2 & \text{Add the second equation} \\
    2x + 2y + z &= 4 & \text{to the third.} \\
    3x + 3y &= 6 & \text{New Equation 2}
\end{align*}
\]

The result is a system of linear equations in two variables.

\[
\begin{align*}
    2x + 2y &= 4 & \text{New Equation 1} \\
    3x + 3y &= 6 & \text{New Equation 2}
\end{align*}
\]

*Solve* the new system by adding \(-3\) times the first equation to \(2\) times the second equation. This produces the identity \(0 = 0\). So, the system has infinitely many solutions.

*Describe* the solution. One way to do this is to divide new Equation 1 by \(2\) to get \(x + y = 2\), or \(y = -x + 2\). Substituting this into original Equation 1 produces \(z = 0\). So, any ordered triple of the form \((x, -x + 2, 0)\) is a solution of the system. For instance, \((0, 2, 0), (1, 1, 0),\) and \((2, 0, 0)\) are all solutions.
**Example 4**  
**Writing and Solving a Linear System**

**Sports** Use a system of equations to model the information in the newspaper article. Then solve the system to find how many swimmers finished in each place.

**Solution**

**Verbal Model**

| 1st-place finishers | 2nd-place finishers | 3rd-place finishers | = Total placers  
|---------------------|---------------------|---------------------|-----------------------
| 5                   | 3                   | 1                   | 24                    

| 1st-place finishers | 2nd-place finishers | 3rd-place finishers | = Total points  
|---------------------|---------------------|---------------------|-----------------------
| 5                   | 3                   | 1                   | 56                    

**Labels**

- 1st-place finishers = \( x \) (people)
- 2nd-place finishers = \( y \) (people)
- 3rd-place finishers = \( z \) (people)
- Total placers = 24 (people)
- Total points = 56 (points)

**Algebraic Model**

\[
\begin{align*}
1x + 2y + 3z &= 24 & \text{Equation 1} \\
5x + 3y + z &= 56 & \text{Equation 2} \\
x + y &= z & \text{Equation 3}
\end{align*}
\]

You now have a system of two equations in two variables.

\[
\begin{align*}
2x + 2y &= 24 & \text{New Equation 1} \\
6x + 4y &= 56 & \text{New Equation 2}
\end{align*}
\]

When you solve this system you get \( x = 4 \) and \( y = 8 \). Substituting these values into original Equation 3 gives you \( z = 12 \). There were 4 first-place finishers, 8 second-place finishers, and 12 third-place finishers.
**GUIDED PRACTICE**

**Vocabulary Check**

1. Give an example of a system of three linear equations in three variables.

**Concept Check**

2. **ERROR ANALYSIS** A student correctly solves a system of equations in three variables and obtains the equation $0 = 3$. The student concludes that the system has infinitely many solutions. Explain the error in the student’s reasoning.

3. Look back at the intersecting planes on page 177. How else can three planes intersect so that the system has infinitely many solutions?

4. Explain how to use the substitution method to solve a system of three linear equations in three variables.

**Skill Check**

**Decide whether the given ordered triple is a solution of the system.**

5. $(1, 4, 2)$
6. $(7, -1, 0)$
7. $(-2, 3, 3)$

- $2x - y + 5z = 12$
- $5x - 2y + z = -13$

- $3x + 2y - z = -7$
- $x + 4y + 3z = 19$

- $-5x + 4y + 2z = -17$
- $-3x + y + 6z = 15$

**Use the indicated method to solve the system.**

8. linear combination
   - $x + 5y - z = 16$
   - $3x - 3y + 2z = 12$
   - $2x + 4y + z = 20$

9. substitution
   - $-2x + y + 3z = -8$
   - $3x + 4y - 2z = 9$
   - $x + 2y + z = 4$

10. any method
    - $9x + 5y - z = -11$
    - $6x + 4y + 2z = 2$
    - $2x - 2y + 4z = 4$

11. **INVESTMENTS** Your aunt receives an inheritance of $20,000. She wants to put some of the money into a savings account that earns 2% interest annually and invest the rest in certificates of deposit (CDs) and bonds. A broker tells her that CDs pay 5% interest annually and bonds pay 6% interest annually. She wants to earn $1000 interest per year, and she wants to put twice as much money in CDs as in bonds. How much should she put in each type of investment?

**PRACTICE AND APPLICATIONS**

**Student Help**

Extra Practice to help you master skills is on p. 944.

**Linear Combination Method** Solve the system using the linear combination method.

12. $3x + 2y - z = 8$
    - $-3x + 4y + 5z = -14$
    - $x - 3y + 4z = -14$

13. $x + 2y + 5z = -1$
    - $2x - y + z = 2$
    - $3x + 4y - 4z = 14$

14. $3x + 2y - 3z = -2$
    - $7x - 2y + 5z = -14$
    - $2x + 4y + z = 6$

15. $5x - 4y + 4z = 18$
    - $-x + 3y - 2z = 0$
    - $4x - 2y + 7z = 3$

16. $x + y - 2z = 5$
    - $x + 2y + z = 8$
    - $2x + 3y - z = 13$

17. $-5x + 3y + z = -15$
    - $10x + 2y + 8z = 18$
    - $15x + 5y + 7z = 9$

**Substitution Method** Solve the system using the substitution method.

18. $-2x + y + 6z = 1$
    - $3x + 2y + 5z = 16$
    - $7x + 3y - 4z = 11$

19. $x - 6y - 2z = -8$
    - $-x + 5y + 3z = 2$
    - $3x - 2y - 4z = 18$

20. $x + y + z = 4$
    - $5x + 5y + 5z = 12$
    - $-x - 4y + z = 9$

21. $x - 3y + 6z = 21$
    - $3x + 2y - 5z = -30$
    - $2x - 5y + 2z = -6$

22. $x + y - 2z = 5$
    - $x + 2y + z = 8$
    - $2x + 3y - z = 1$

23. $2x - 3y + z = 10$
    - $y + 2z = 13$
    - $z = 5$
CHOOSING A METHOD  Solve the system using any algebraic method.

24. \(2x - 2y + z = 3\)
   \(5y - z = -31\)
   \(x + 3y + 2z = -21\)

25. \(17x - y + 2z = -9\)
   \(x + y - 4z = 8\)
   \(3x - 2y - 12z = 24\)

26. \(-2x + y + z = -2\)
   \(5x + 3y + 3z = 71\)
   \(4x - 2y - 3z = 1\)

27. \(x - 9y + 4z = 1\)
   \(-4x + 18y - 8z = -6\)
   \(2x + y - 4z = -3\)

28. \(2x + y + 2z = 7\)
   \(2x - y + 2z = 1\)
   \(5x + y + 5z = 13\)

29. \(7x - 3y + 4z = -14\)
   \(8x + 2y - 24z = 18\)
   \(6x - 10y + 8z = -24\)

30. \(12x + 6y + 7z = -35\)
   \(7x - 5y - 6z = 200\)
   \(x + y = -10\)

31. \(7x - 10y + 8z = -50\)
   \(-2x - 5y + 12z = -90\)
   \(3x + 4y + 4z = 26\)

32. \(-2x - 3y - 6z = -26\)
   \(5x + 5y + 4z = 24\)
   \(3x + 4y - 5z = -40\)

33. \(3x + 3y + z = 30\)
   \(10x - 3y - 7z = 17\)
   \(-6x + 7y + 3z = -49\)

34. Field Trip  You and two friends buy snacks for a field trip. Using the information given in the table, determine the price per pound for mixed nuts, granola, and dried fruit.

<table>
<thead>
<tr>
<th>Shopper</th>
<th>Mixed nuts</th>
<th>Granola</th>
<th>Dried fruit</th>
<th>Total price</th>
</tr>
</thead>
<tbody>
<tr>
<td>You</td>
<td>1 lb</td>
<td>(\frac{1}{2}) lb</td>
<td>(\frac{1}{2}) lb</td>
<td>$5.97</td>
</tr>
<tr>
<td>Kenny</td>
<td>1(\frac{1}{2}) lb</td>
<td>(\frac{1}{4}) lb</td>
<td>(\frac{3}{2}) lb</td>
<td>$9.22</td>
</tr>
<tr>
<td>Vanessa</td>
<td>(\frac{1}{3}) lb</td>
<td>1(\frac{1}{2}) lb</td>
<td>2 lb</td>
<td>$10.96</td>
</tr>
</tbody>
</table>

35. Track Meet  Use a system of linear equations to model the data in the following newspaper article. Solve the system to find how many athletes finished in each place.

**Lawrence High** prevailed in Saturday’s track meet with the help of 20 individual-event placers earning a combined 68 points. A first-place finish earns 5 points, a second-place finish earns 3 points, and a third-place finish earns 1 point. Lawrence had a strong second-place showing, with as many second-place finishers as first- and third-place finishers combined.

36. Chinese Restaurant  Jeanette, Raj, and Henry go to a Chinese restaurant for lunch and order three different luncheon combination platters. Jeanette orders 2 portions of fried rice and 1 portion of chicken chow mein. Raj orders 1 portion of fried rice, 1 portion of chicken chow mein, and 1 portion of sautéed broccoli. Henry orders 1 portion of sautéed broccoli and 2 portions of chicken chow mein. Jeanette’s platter costs $5, Raj’s costs $5.25, and Henry’s costs $5.75. How much does 1 portion of chicken chow mein cost?
37. Write a system of equations for the three combinations of furniture.

38. What is the price of each piece of furniture?

39. Social Studies Connection For several political parties, the table shows the approximate percent of votes for the party’s presidential candidate that were cast in 1996 by voters in two regions of the United States. Write and solve a system of equations to find the total number of votes for each party (Democrat, Republican, and Other). Use the fact that a total of about 100 million people voted in 1996. Source: Statistical Abstract of the United States

<table>
<thead>
<tr>
<th>Region</th>
<th>Democrat (%)</th>
<th>Republican (%)</th>
<th>Other parties (%)</th>
<th>Total voters (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northeast</td>
<td>20</td>
<td>15</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>South</td>
<td>30</td>
<td>35</td>
<td>25</td>
<td>31.5</td>
</tr>
</tbody>
</table>

40. Going in Reverse Which values should be given to \(a\), \(b\), and \(c\) so that the linear system shown has \((-1, 2, -3)\) as its only solution?

\[
\begin{align*}
x + 2y - 3z &= a \\
-x - y + z &= b \\
2x + 3y - 2z &= c
\end{align*}
\]

41. Critical Thinking Write a system of three linear equations in three variables that has the given number of solutions.

a. one solution
b. no solution
c. infinitely many solutions

42. Multi-Step Problem You have $25 to spend on picking 21 pounds of three different types of apples in an orchard. The Empire apples cost $1.40 per pound, the Red Delicious apples cost $1.10 per pound, and the Golden Delicious apples cost $1.30 per pound. You want twice as many Red Delicious apples as the other two kinds combined.

a. Write a system of equations to represent the given information.

b. How many pounds of each type of apple should you buy?

c. Writing Create your own situation in which you are buying three different types of fruit. State the total amount of fruit you need, the price of each type of fruit, the amount of money you have to spend, and the desired ratio of one type of fruit to the other two types. Write a system of equations representing your situation. Then solve your system to find the number of pounds of each type of fruit you should buy.

43. \(w + x + y + z = 6\)

44. \(2w - x + 5y + z = -3\)

\(3w - x + y - z = -3\)

\(2w + 2x - 2y + z = 4\)

\(2w - x - y + z = -4\)

\(3w + 2x + 2y - 6z = -32\)

\(w + 3x + 3y - z = -47\)

\(5w - 2x - 3y + 3z = 49\)
PERFORMING AN OPERATION Perform the indicated operation. (Review 1.1 for 4.1)

45. \(-10 + 21\)  
46. \(15 - (-1)\)  
47. \(12 \cdot 7\)

48. \(-2 - (-20)\)  
49. \(-9 + (-7)\)  
50. \(-8(-6)\)

51. \(-\frac{1}{2} + \frac{4}{5}\)  
52. \(-\frac{1}{3}\left(\frac{2}{7}\right)\)  
53. \(\frac{3}{4} - 3\)

SOLVING AND GRAPHING Solve the inequality. Then graph your solution. (Review 1.7)

54. \(|11 - x| < 20\)  
55. \(|2x + 3| \geq 26\)  
56. \(|18 + \frac{1}{2}x| \geq 10\)

57. \(|7 + 8x| > 5\)  
58. \(|5 - x| < 10\)  
59. \(|3x - 1| \leq 30\)

60. \(|-3x + 6| \geq 12\)  
61. \(|6x + 4| < 40\)  
62. \(|15 - 3x| > 3\)

PLOTTING POINTS Plot the ordered triple in a three-dimensional coordinate system. (Review 3.5)

63. \((3, 6, 0)\)  
64. \((-3, -6, -4)\)  
65. \((-5, 9, 2)\)

66. \((-9, 4, -7)\)  
67. \((6, -2, -6)\)  
68. \((-8, 5, -6)\)

69. \((0, -3, -3)\)  
70. \((2, 2, -2)\)  
71. \((-4, -7, -3)\)

Quiz 3

Self-Test for Lessons 3.5 and 3.6

Sketch the graph of the equation. Label the points where the graph crosses the x-, y-, and z-axes. (Lesson 3.5)

1. \(2x + 5y + 3z = 15\)  
2. \(x + 4y + 16z = 8\)  
3. \(3x + y + z = 10\)

4. \(3x + 12y + 6z = 9\)  
5. \(5x - 2y + z = 15\)  
6. \(-x + 9y - 3z = 18\)

Write the linear equation as a function of \(x\) and \(y\). Then evaluate the function for the given values. (Lesson 3.5)

7. \(-x + \frac{1}{2}y + 3z = 18, f(2, 0)\)  
8. \(4x + 8y - 8z = -16, f(-4, 4)\)

9. \(20x - 3y - z = 15, f(3, -7)\)  
10. \(-2x + y + 6z = 24, f(12, 7)\)

Solve the system using any algebraic method. (Lesson 3.6)

11. \(2x + 4y + 3z = 10\)  
12. \(3x - 2y + 3z = 11\)  
13. \(x - 2y + 3z = -9\)

14. \(3x - y + 6z = 15\)  
15. \(5x + 2y - 2z = 4\)  
16. \(2x + 5y + z = 10\)

17. \(5x + 2y - z = 25\)  
18. \(-x + y + z = -7\)  
19. \(3x - 6y + 9z = 12\)

14. STATE ORCHESTRA Fifteen band members from your school were selected to play in the state orchestra. Twice as many students who play a wind instrument were selected as students who play a string or percussion instrument. Of the students selected, one fifth play a string instrument. How many students playing each type of instrument were selected to play in the state orchestra? (Lesson 3.6)
Chapter Summary

WHAT did you learn?

Solve systems of linear equations in two variables.
  • by graphing (3.1)
  • using algebraic methods (3.2)

Graph and solve systems of linear inequalities. (3.3)

Solve linear programming problems. (3.4)

Graph linear equations in three variables. (3.5)

Model real-life problems with functions of two variables. (3.5)

Solve systems of linear equations in three variables. (3.6)

Identify the number of solutions of a linear system. (3.1, 3.2, 3.6)

Solve real-life problems.
  • using a system of linear equations (3.1, 3.2, 3.6)
  • using a system of linear inequalities (3.3, 3.4)

WHY did you learn it?

Plan a vacation within a budget. (p. 141)

Find the weights of atoms in a molecule. (p. 153)

Describe conditions that will satisfy nutritional requirements of wildlife. (p. 161)

Plan a meal that minimizes cost while satisfying nutritional requirements. (p. 167)

Find the volume of a geometric figure graphed in a three-dimensional coordinate system. (p. 174)

Evaluate advertising costs of a commercial. (p. 175)

Use regional data to find the number of voters for different political parties in the United States. (p. 183)

See if a bus catches up to another one before arriving at a common destination. (p. 144)

Find the break-even point of a business. (p. 153)

Display possible sale prices for shoes. (p. 161)

How does Chapter 3 fit into the BIGGER PICTURE of algebra?

Linear algebra is an important branch of mathematics that begins with solving linear systems. It has widespread applications to other areas of mathematics and to real-life problems, especially in business and the sciences. You will continue your study of linear algebra in the next chapter with matrices.

Did you recognize when new skills related to previously learned skills?

The two-column list you made, following the Study Strategy on page 138, may resemble this one.
3.1 Solving Linear Systems by Graphing

You can solve a system of two linear equations in two variables by graphing.

\[
\begin{align*}
x + 2y &= -4 \\
3x + 2y &= 0
\end{align*}
\]

Equation 1
Equation 2

From the graph, the lines appear to intersect at \((2, -3)\). You can check this algebraically as follows.

\[
\begin{align*}
2 + 2(-3) &= -4 \checkmark & \text{Equation 1 checks.} \\
3(2) + 2(-3) &= 0 \checkmark & \text{Equation 2 checks.}
\end{align*}
\]

Graph the linear system and tell how many solutions it has. If there is exactly one solution, estimate the solution and check it algebraically.

1. \(x + y = 2\)
   \(-3x + 4y = 36\)

2. \(x - 5y = 10\)
   \(-2x + 10y = -20\)

3. \(2x - y = 5\)
   \(2x + 3y = 9\)

4. \(y = \frac{1}{3}x\)
   \(y = \frac{1}{3}x - 2\)

3.2 Solving Linear Systems Algebraically

\[x - 4y = -25\]
\(2x + 12y = 10\)

\[\begin{align*}
x - 4y &= -25 \\
x &= 4y - 25 \\
2(4y - 25) + 12y &= 10 \\
y &= 3
\end{align*}\]

When you substitute \(y = 3\) into one of the original equations, you get \(x = -13\).
You can also use the linear combination method to solve a system of equations algebraically.

1. Multiply the first equation by 3 and add to the second equation. Solve for \( x \).
   
   \[
   \begin{align*}
   x - 4y &= -25 \\
   2x + 12y &= 10 \\
   \hline
   5x &= -65
   \end{align*}
   \]

   \[ x = -13 \]

2. Substitute \( x = -13 \) into the original first equation and solve for \( y \).
   
   \[
   \begin{align*}
   -13 - 4y &= -25 \\
   -4y &= -12 \\
   y &= 3
   \end{align*}
   \]

Solve the system using any algebraic method.

5. \( 9x - 5y = -30 \)
   
6. \( x + 3y = -2 \)

7. \( 2x + 3y = -7 \)

8. \( 3x + 3y = 0 \)

---

**Graphing and Solving Systems of Linear Inequalities**

You can use a graph to show all the solutions of a system of linear inequalities.

\[
\begin{align*}
 x &\geq 0 \\
y &\geq 0 \\
x + 2y &< 10
\end{align*}
\]

Graph each inequality. The graph of the system is the region common to all of the shaded half-planes and includes any solid boundary line.

**Graph the system of linear inequalities.**

9. \( y < -3x + 3 \)
   
   \( y > x - 1 \)

10. \( x \geq 0 \)
    
    \( y \geq 0 \)
    
    \( -x + 2y < 8 \)

11. \( x \geq -2 \)
    
    \( x \leq 5 \)
    
    \( y \geq -1 \)
    
    \( y \leq 3 \)

12. \( x + y \leq 8 \)
    
    \( 2x - y > 0 \)
    
    \( y \leq 4 \)

---

**Linear Programming**

You can find the minimum and maximum values of the objective function \( C = 6x + 5y \) subject to the constraints graphed below. They must occur at vertices of the feasible region.

At \( (0, 0) \): \( C = 6(0) + 5(0) = 0 \) \(-\) Minimum

At \( (0, 3) \): \( C = 6(0) + 5(3) = 15 \)

At \( (5, 2) \): \( C = 6(5) + 5(2) = 40 \)

At \( (7, 0) \): \( C = 6(7) + 5(0) = 42 \) \(+\) Maximum
3.5

**GRAPHING LINEAR EQUATIONS IN THREE VARIABLES**

**EXAMPLE** You can sketch the graph of an equation in three variables in a three-dimensional coordinate system.

To graph $3x + 4y - 3z = 12$, find $x$-, $y$-, and $z$-intercepts.

If $y = 0$ and $z = 0$, then $x = 4$. Plot $(4, 0, 0)$.

If $x = 0$ and $z = 0$, then $y = 3$. Plot $(0, 3, 0)$.

If $x = 0$ and $y = 0$, then $z = -4$. Plot $(0, 0, -4)$.

Draw the plane that contains $(4, 0, 0)$, $(0, 3, 0)$, and $(0, 0, -4)$.

Sketch the graph of the equation. Label the points where the graph crosses the $x$-, $y$-, and $z$-axes.

17. $x + y + z = 5$

18. $5x + 3y + 6z = 30$

19. $3x + 6y - 4z = -12$

3.6

**SOLVING SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES**

**EXAMPLE** You can use algebraic methods to solve a system of linear equations in three variables. First rewrite it as a system in two variables.

1. Add the first and second equations.

   \[
   \begin{align*}
   x - 3y + z &= 22 \\
   2x - 2y - z &= -9 \\
   x + y + 3z &= 24 \\
   \end{align*}
   \]

   \[
   \begin{align*}
   x - 3y + z &= 22 \\
   2x - 2y - z &= -9 \\
   3x - 5y &= 13 \\
   \end{align*}
   \]

2. Multiply the second equation by 3 and add to the third equation.

   \[
   \begin{align*}
   6x - 6y - 3z &= -27 \\
   x + y + 3z &= 24 \\
   \end{align*}
   \]

   \[
   \begin{align*}
   7x - 5y &= -3 \\
   \end{align*}
   \]

3. Solve the new system.

   \[
   \begin{align*}
   3x - 5y &= 13 \\
   -7x + 5y &= 3 \\
   \end{align*}
   \]

   \[
   \begin{align*}
   -4x &= 16 \\
   \end{align*}
   \]

   \[
   \begin{align*}
   x &= -4 \quad \text{and} \quad y = -5 \\
   \end{align*}
   \]

When you substitute $x = -4$ and $y = -5$ into one of the original equations, you get the value of the last variable: $z = 11$.

Solve the system using any algebraic method.

20. $x + 2y - z = 3$

   \[
   \begin{align*}
   -x + y + 3z &= -5 \\
   3x + y + 2z &= 4 \\
   \end{align*}
   \]

21. $2x - 4y + 3z = 1$

   \[
   \begin{align*}
   6x + 2y + 10z &= 19 \\
   -2x + 5y - 2z &= 2 \\
   \end{align*}
   \]

22. $x + y + z = 3$

   \[
   \begin{align*}
   x + y - z &= 3 \\
   2x + 2y + z &= 6 \\
   \end{align*}
   \]
Graph the linear system and tell how many solutions it has. If there is exactly one solution, estimate the solution and check it algebraically.

1. \( x + y = 1 \)            2. \( y = -\frac{1}{3}x + 4 \)            3. \( y = 2x + 2 \)
   \( 2x - 3y = 12 \)            \( y = 6 \)            \( y = 2x - 3 \)

4. \( \frac{1}{2}x + 5y = 2 \)
   \( -x - 10y = -4 \)

Solve the system using any algebraic method.

5. \( 3x + 6y = -9 \)            6. \( x - y = -5 \)            7. \( 7x + y = -17 \)            8. \( 8x + 3y = -2 \)
   \( x + 2y = -3 \)            \( x + y = 11 \)            \( 3x - 10y = 24 \)            \( -5x + y = -3 \)

Graph the system of linear inequalities.

9. \( 2x + y \geq 1 \)
   \( x \leq 3 \)
10. \( x \geq 0 \)
    \( y < x \)
    \( y > -x \)
11. \( x + 2y \geq -6 \)
    \( x + 2y \leq 2 \)
    \( 2x - y \geq 5 \)
    \( x \geq -2 \)
12. \( x + y < 7 \)

Find the minimum and maximum values of the objective function subject to the given constraints.

13. Objective function: \( C = 7x + 4y \)
    Constraints: \( x \geq 0 \)
                 \( y \geq 0 \)
                 \( 4x + 3y \leq 24 \)
14. Objective function: \( C = 3x + 4y \)
    Constraints: \( x + y \leq 10 \)
                 \( -x + y \leq 5 \)
                 \( 2x + 4y \leq 32 \)

Plot the ordered triple in a three-dimensional coordinate system.

15. \((-1, 3, 2)\)
16. \((0, 4, -2)\)
17. \((-5, -1, 2)\)
18. \((6, -2, 1)\)

Sketch the graph of the equation. Label the points where the graph crosses the \(x\)-, \(y\)-, and \(z\)-axes.

19. \(2x + 3y + 5z = 30\)
20. \(4x + y + 2z = 8\)
21. \(3x + 12y - 6z = 24\)

22. Write the linear equation \(2x - 5y + z = 9\) as a function of \(x\) and \(y\). Then evaluate the function when \(x = 10\) and \(y = 3\).

Solve the system using any algebraic method.

23. \(x + 2y - 6z = 23\)
24. \(x + y + 2z = 1\)
25. \(x + 3y - z = 1\)
   \(x + 3y + z = 4\)
   \(x - y + z = 0\)
   \(-4x - 2y + 5z = 16\)
26. \(2x + 5y - 4z = 24\)
   \(3x + 3y + 6z = 4\)
   \(7x + 10y + 6z = -15\)

26. **Craft Supplies** You are buying beads and string to make a necklace.
    The string costs $1.50, a package of 10 decorative beads costs $.50, and a package of 25 plain beads costs $.75. You can spend only $7.00 and you need 150 beads. How many packages of each type of bead should you buy?

27. **Business** An appliance store manager is ordering chest and upright freezers. One chest freezer costs $250 and delivers a $40 profit. One upright freezer costs $400 and delivers a $60 profit. Based on previous sales, the manager expects to sell at least 100 freezers. Total profit must be at least $4800. Find the least number of each type of freezer the manager should order to minimize costs.
Chapter Standardized Test

TEST-TAKING STRATEGY If you find yourself spending too much time on one test question and getting frustrated, move on to the next question. You can revisit a difficult problem later with a fresh perspective.

1. **Multiple Choice** Which ordered pair is a solution of the following system of linear equations?

   \[ \begin{align*}
   2x - 5y &= -12 \\
   -x + 4y &= 9
   \end{align*} \]

   A (−6, 0)  B (3, 3)  C (−1, 2)  D (−9, 0)  E (2, 2)

2. **Multiple Choice** How many solutions does the following system have?

   \[ \begin{align*}
   8x - 4y &= 20 \\
   2x - y &= 5
   \end{align*} \]

   A 0  B 1  C 2  D 4  E infinitely many

3. **Multiple Choice** A total of $6500 is invested in two funds. One fund pays 4% interest annually and the other fund pays 6% interest annually. The combined annual interest earned is $350. How much of the $6500 is invested in one of the funds?

   A $2000  B $2500  C $3250  D $4000  E $5500

4. **Multiple Choice** Which ordered pair is not a solution of the following system of linear inequalities?

   \[ \begin{align*}
   x &= \geq -2 \\
   y &= \geq -3 \\
   y &= < 3x + 3
   \end{align*} \]

   A (4, −3)  B (0, 0)  C (1, 6)  D (5, 17)  E (−1, −1)

5. **Multiple Choice** What is the minimum value of the objective function \( C = 4x + 3y \) subject to the following constraints?

   \[ \begin{align*}
   x &\geq 0 \\
   y &\geq 0 \\
   2x + 3y &\leq 18 \\
   3x + y &\geq 6
   \end{align*} \]

   A 0  B 2  C 8  D 18  E 36

6. **Multiple Choice** Which linear equation is graphed below?

   A \( x - 2y - z = 4 \)  B \( x - 2y + z = -4 \)  C \( x + 2y - z = -4 \)  D \( x + 2y - z = 4 \)  E \( -x + 2y + z = 4 \)

7. **Multiple Choice** At which point does the graph of \( 15x - 6y - 3z = 30 \) cross the y-axis?

   A (0, −6, 0)  B (2, 0, 0)  C (0, −3, 0)  D (0, 0, −10)  E (0, −5, 0)

8. **Multiple Choice** Which ordered triple is a solution of the following linear system?

   \[ \begin{align*}
   2x + 5y + 3z &= 10 \\
   3x - y + 4z &= 8 \\
   5x - 2y + 7z &= 12
   \end{align*} \]

   A (7, 1, −3)  B (7, −1, −3)  C (7, 1, 3)  D (7, −1, 3)  E (−7, 1, −3)

9. **Multiple Choice** A cashier at a restaurant made the chart below for popular lunch combinations. What is the individual price of soup?

<table>
<thead>
<tr>
<th>Lunch Combinations</th>
<th>Price (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soup + Salad = 4.25</td>
<td>A $1.50</td>
</tr>
<tr>
<td>Soup + Sandwich = 4.75</td>
<td>B $1.75</td>
</tr>
<tr>
<td>Salad + Sandwich = 5.50</td>
<td>C $2.25</td>
</tr>
<tr>
<td></td>
<td>D $2.50</td>
</tr>
<tr>
<td></td>
<td>E $3.00</td>
</tr>
</tbody>
</table>
QUANTITATIVE COMPARISON  In Exercises 10 and 11, choose the statement that is true about the given quantities.

A The quantity in column A is greater.
B The quantity in column B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the given information.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x, y) = \frac{1}{5}(20 - 2x + y) ), ( f(4, 8) )</td>
<td>( f(x, y) = \frac{1}{5}(20 - 2x + y) ), ( f(-1, 3) )</td>
</tr>
<tr>
<td>( f(x, y) = \frac{1}{2}(10 + 4x - 3y) ), ( f(-2, -1) )</td>
<td>( f(x, y) = \frac{1}{2}(10 + 4x - 3y) ), ( f(2, 1) )</td>
</tr>
</tbody>
</table>

12. **Multi-Step Problem**  Use the following system of linear equations.

\[
\begin{align*}
  x - 2y &= 2 \\
  5x - 4y &= -8
\end{align*}
\]

Equation 1
Equation 2

a. Solve the system by graphing.
b. Solve the system using the substitution method. Show your work.
c. Solve the system using the linear combination method. Show your work.
d. **Writing**  Which method do you prefer for solving this system? Explain.

13. **Multi-Step Problem**  Write an equation which, when paired with 
\(-2x + 3y = 12\) to form a system, has the given number of solutions.

a. exactly one solution
b. no solution
c. infinitely many solutions
d. **Writing**  Explain how you wrote each of the equations in parts (a)–(c).

14. **Multi-Step Problem**  The cholesterol in your blood is necessary, but too much cholesterol can lead to health problems. A blood cholesterol test gives three readings: LDL “bad” cholesterol, HDL “good” cholesterol, and total cholesterol (LDL + HDL). It is recommended that your LDL cholesterol be less than 130 milligrams per deciliter, HDL cholesterol be at least 35 milligrams per deciliter, and total cholesterol be no more than 200 milligrams per deciliter.

a. Write a system of three linear inequalities for the recommended cholesterol readings. Let \( x \) represent HDL cholesterol and \( y \) represent LDL cholesterol.
b. Graph the system. Label any vertices of the solution region.
c. Are the cholesterol readings at the right within recommendations?
d. Give an example of blood cholesterol test results in which the LDL cholesterol is too high, but HDL and total cholesterol readings are fine. Write a system of linear inequalities to describe all the examples of this type.
e. Another recommendation is that the ratio of total cholesterol to HDL cholesterol be less than 4. Find a point in your solution region from part (b) that meets this recommendation and show that it does.
Plot the numbers on a number line. Write the numbers in increasing order. (1.1)

1. 0, π, \(\frac{3}{4}\), \(-\frac{3}{2}\), 4
2. \(\frac{5}{2}\), \(-\frac{1}{10}\), \(-2\), \(\sqrt{5}\), 1.9
3. \(-4.25\), \(-\frac{16}{3}\), \(-\sqrt{9}\), \(-0.4\), \(-1\)

Identify the property shown. (1.1)

4. \(8 \cdot \frac{1}{8} = 1\)
5. \(-1(9 + 7) = (-1)9 + (-1)7\)
6. \(-6 \cdot (-3 \cdot 4) = (-6 \cdot (-3)) \cdot 4\)

Evaluate the expression. (1.2)

7. \(12 / 2 - 4 \cdot 7\)
8. \(-8 + 3(1 - 5)\)
9. \(17 - 2^4 + 8 + 1\)
10. \(-2(16 + 7) + 10\)

Simplify the expression. (1.2)

11. \(18a + 7a - 9a + 11\)
12. \(10x - (4y - x) + y\)
13. \(6(n^2 - n) - 5n^2 + 8n\)

Solve the equation. (1.3, 1.7)

14. \(5\sqrt{8x} - 9 = 21\)
15. \(-75 = 9x - 3\)
16. \(4(2x - 1) = -20\)
17. \(3 - x = 5x + 27\)

Solve the inequality. Then graph the solution. (1.6, 1.7)

25. \(14 - 5x > -6\)
26. \(1 \leq x - 13 \leq 20\)
27. \(3x - 2 \leq 0\) or \(x + 6 > 8\)
28. \(|x - 7| \leq 1\)
29. \(|7x - 9| \geq 12\)
30. \(|\frac{1}{4}x + 3| > 5\)
31. \(|-5x| < 10\)

Graph the relation. Then tell whether the relation is a function. (2.1)

32. \[
\begin{array}{c|ccccc}
\text{x} & 2 & -4 & 2 & -1 & 0 \\
\hline
\text{y} & 1 & 0 & 5 & -1 & 3
\end{array}
\]
33. \[
\begin{array}{c|ccccc}
\text{x} & -3 & -1 & 1 & 3 & 5 \\
\hline
\text{y} & 1 & 0 & -1 & -2 & -3
\end{array}
\]

Graph in a coordinate plane. (2.1, 2.3, 2.6–2.8)

34. \(y = -2x + 5\)
35. \(x - 3y = 6\)
36. \(y = 2\)
37. \(x = -4\)
38. \(y > \frac{2}{5}x - 2\)
39. \(y \leq -1\)
40. \(4x + 3y \leq 24\)
41. \(y > -x\)
42. \(f(x) = 4|x|\)
43. \(f(x) = |x| - 3\)
44. \(f(x) = 2|x + 2|\)
45. \(f(x) = -|x - 5| + 1\)
46. \(f(x) = \begin{cases} 2x, & \text{if } x \leq 0 \\ -2x, & \text{if } x > 0 \end{cases}\)
47. \(f(x) = \begin{cases} \frac{1}{2}x + 1, & \text{if } x \leq -2 \\ x + 1, & \text{if } x > -2 \end{cases}\)
48. \(f(x) = \begin{cases} 4, & \text{if } -5 \leq x < 0 \\ -4, & \text{if } 0 \leq x \leq 5 \end{cases}\)

Graph the system. Describe the solution(s). (3.1, 3.3)

49. \(4x - 2y = 8\) \(4x + y = 2\)
50. \(y = x\) \(y = x - 3\) \(y = x + 5\)
51. \(2x - y > 1\) \(x < 3\)
52. \(x \geq 0\) \(y \geq 0\) \(x + y \leq 8\)
Tell whether the lines are perpendicular, parallel, or neither. (2.2)

53. Line 1: through (0, 7) and (3, 6)
   Line 2: through (−2, −9) and (0, −3)
54. Line 1: through (−6, −3) and (0, 1)
   Line 2: through (0, −5) and (4, −2)

Write an equation of the line with the given characteristics. (2.4)

55. slope: −3, y-intercept: 7
56. vertical line through (2, 5)
57. x-intercept: −2, y-intercept: 1

Evaluate the function for the given value(s). (2.1, 2.7, 2.8, 3.5)

58. \( f(x) = 5x - 17, \ f(−3) \)
59. \( f(x) = x^2 - 2x + 11, \ f(2) \)
60. \( f(x) = \begin{cases} x - 4, & \text{if } x \leq 0 \\ x + 2, & \text{if } x > 0 \end{cases}, \ f(−2) \)
61. \( f(x) = -|12 - 8x|, \ f(1) \)
62. \( f(x, y) = 8x - 5y, \ f(3, -2) \)
63. \( f(x, y) = 2(-x + y), \ f(-1, 0) \)

Solve the system using any algebraic method. (3.2, 3.6)

64. \(-x + 5y = 8 \)
   \(-3x + 15y = 24 \)
65. \( x - 3y = 7 \)
   \( 2x + y = 7 \)
66. \( x + y - z = 7 \)
   \( -x + 2y + 2z = 3 \)
   \( 3x - y - z = 1 \)
67. \( 2x + y + z = 4 \)
   \( x - y - 2z = -9 \)
   \( 2x - y + z = 6 \)

Graph in a three-dimensional coordinate system. (3.5)

68. \((1, -4, 2)\)
69. \((-2, 3, -5)\)
70. \(x + 2y + 3z = 6\)
71. \(10x + 4y + 5z = 20\)

72. 

73. 

74. 

75. 

76. 

77. 

Source: Statistical Abstract of the United States

Cumulative Practice
**PROJECT**

**Applying Chapters 1–3**

**Drawing with Linear Perspective**

**OBJECTIVE** Use linear equations to represent a drawing made with linear perspective.

**Materials:** graph paper, ruler

During the Renaissance, artists turned to mathematics to develop *perspective*, a method for realistically depicting a three-dimensional object on a two-dimensional surface. A drawing with linear perspective has all slanted lines converging toward a point or points on the horizon. These points are called *vanishing points*. The painting below and on the left has all slanted lines converging toward a single vanishing point at the far end of the road. The painting on the right has all slanted lines converging toward one of two vanishing points, one on either side of the building.

1. Use the $x$-axis as the horizon. Select two points equidistant from the origin and on the $x$-axis as the vanishing points. Draw a vertical segment to represent the front edge of the object—in this case, the front edge of a building.

2. To draw the left wall of the building, draw segments from the endpoints of the front edge toward the vanishing point on the left. Connect the segments with a vertical line to represent the end of the wall.

3. Continue drawing slanted lines that are to the left of the front edge toward the vanishing point on the left, and lines that are to the right of the front edge toward the vanishing point on the right.

*The Avenue at Middelharnis, painted in 1689 by Meindert Hobbema*

*Corner of George and Hunter Streets, Sydney, painted in 1849 by A. Tornig*
INVESTIGATION

1. Choose an object that has many parallel edges, such as a building, courtyard, or computer. Use the method given on the previous page to draw the object in two-point perspective.

2. Experiment with using a lower or higher horizon line, as well as vanishing points that are farther apart or closer together, until your drawing has the look you want. How does the placement of the horizon line and the vanishing points affect the way your drawing looks?

3. Write an equation for each line in your drawing. Include the domain to indicate the length of the line. For example, the upper left edge of the building on the previous page is defined by \( y = \frac{2}{3}x + 4 \) for \(-4 \leq x \leq 0\).

PRESENT YOUR RESULTS

Write a report to present your results.

- Include your drawing and any preliminary sketches you did.
- Include your answers to Exercises 1–3 above.
- Write a set of instructions for how to draw the object just as you have drawn it. Include the equations you wrote.
- Tell how this project has helped you mathematically.

Test your results.

- Trade drawing instructions with a partner (do not trade actual drawings). Follow the instructions to create your partner’s drawing.
- Compare your drawing with the original.

EXTENSION

Another way to suggest a three-dimensional object on a two-dimensional surface is to add shadowing. Select a point for a light source and decide where the shadows cast by your object would fall. Write a system of linear inequalities to indicate each shaded region. Add these to your report.

System of inequalities for the building’s shadow:

\[
\begin{align*}
y & \leq \frac{1}{5}x - 2 \\
y & \geq -\frac{1}{5}x - 2 \\
y & \geq \frac{3}{5}x - 6 \\
y & \leq -\frac{3}{35}x - \frac{6}{7}
\end{align*}
\]